MA 114H Calculus II, Fall 2014
Written Assignment #1
Due **Friday, 17 October 2014**, at the beginning of lecture.

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, careful explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. Your textbook provides examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

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1. (5 points) The goal of this exercise is to derive the Taylor polynomial of degree n, $T_n(x)$, and an upper bound on the remainder, $R_n(x)$, for the function $f(x) = (1+x)^\alpha$, where α is any real number. When $\alpha = n \in N$, a positive integer, this is the familiar Binomial Theorem.

   (a) (3 points) Write the n^{th}-degree Taylor polynomial $T_n(x)$ for $(1+x)^\alpha$ about $c = 0$. Express your answer using the general binomial coefficient described below.

   (b) (2 points) Let $R_n(x) = f(x) - T_n(x)$, where $f(x) = (1+x)^\alpha$ Write the the formula for $R_n(x)$. When $\alpha < 0$, give an upper bound for $|R_n(x)|$ depending on α, n, and x.

2. (5 points) Einstein's special theory of relativity gives the total energy of a particle of mass $m > 0$ moving with speed $v \geq 0$ as

$$E(v) = mc^2 \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2},$$

   where the constant c is the speed of light. When $v = 0$, the energy $E_0 = E(0) = mc^2$ is the rest energy of the particle.

   (a) (3 points) Suppose $v << c$. Write an expansion of order n and remainder for the total energy $E(v) = T_n(v) + R_n(v)$ for any positive integer n.

   (b) (2 points) The difference $E_{\text{kin}}(v) = E(v) - E_0$ is the kinetic energy of the particle. How small must the speed of the particle be relative to the speed of light c in order that the classical expression $E_{\text{kin}}(v) = \frac{1}{2}mv^2$ describes the kinetic energy to one part in $10^{-6}mc^2$ (include the rest energy)? Since $c = 3 \times 10^8 m/sec$, give an estimate on v so this holds. Can you neglect relativistic effects for a spaceship moving at $11,200 m/sec$, the escape velocity for the earth?

NOTE: We will always define $\beta! = \beta(\beta-1)(\beta-2)(\beta-3)\cdots$ for any real number β. If β isn't a positive integer, this product never ends. We can write the general Binomial Theorem using the general binomial coefficient:

$$\binom{\beta}{j} = \frac{\beta!}{j!(\beta-j)!} = \frac{\beta(\beta-1)(\beta-2)(\beta-3)\cdots(\beta-(j-1))}{j!}$$

and the numerator has only finitely-many factors.