· MA 114 H
· F2014
WA 3

WA3 Solution

Let $y(t)$ be the number of miles cleared by the plow since noon. We set $t=0$ to be noon. Then the snow started at time $t=-a$, for some $a>0$. We need to find a.

Since the snow is steady, it accumulates at a constant rate $R>0$ so the depth at time t is proportional to $(t+a)$. When $t=-a$, the depth is 0. The rate of miles plowed per unit time $\frac{dy}{dt}$ is proportional to $\frac{1}{R(t+a)}$. The more volume to move, the slower the progress!

We get $\frac{dy}{dt} = \frac{C_1}{R(t+a)}$. Solving this

$$\boxed{y(t) = \left(\frac{C_1}{R}\right)\ln(t+a) + C_2}$$

The initial condition: $y(t=0)=0$ so $C_2 = -\frac{C_1}{R}\ln a$

This gives $y(t) = \frac{C_1}{R}\ln\left(\frac{t+a}{a}\right)$, $t \geq 0$, $y(0) = 0$.

We now impose the conditions: After 2 hrs. the plow covers 2 miles: $y(2) = 2$ and after 2 more hours, it covers 1 mile more so $y(4) = 2+1 = 3$. Substitute these into the solution:

$$y(2) = 2 = \left(\frac{C_1}{R}\right)\ln[(2+a)/a]$$

$$y(4) = 3 = \left(\frac{C_1}{R}\right)\ln[(4+a)/a]$$

Take the ratio:

$$\frac{y(4)}{y(2)} = \frac{3}{2} = \frac{\ln[(4+a)/a]}{\ln[(2+a)/a]}$$

You can solve this for a:

$$\ln\left(\frac{2+a}{a}\right)^{3/2} = \ln\left(\frac{4+a}{a}\right)$$

$$\Rightarrow \left(\frac{2+a}{a}\right)^3 = \left(\frac{4+a}{a}\right)^2$$

$$\Rightarrow (2+a)^3 = a(4+a)^2$$

Reduce to: $a^2 + 2a - 4 = 0$

Solutions: $a_\pm = -1 \pm \sqrt{5}$

Since $a > 0$ choose $a = -1 + \sqrt{5} \approx 1.24$ hrs before noon!

Snow started at approximately 10:46 AM