1. Find the most general solution to the ODE:
   \[ 4y'(t) + y(t) = 8t. \]

2. Find the most general solution to the ODE:
   \[ x \frac{dy}{dx} + y = \sin x. \]

3. Find the most general solution to the ODE:
   \[ 3 \frac{dy}{dx} - y = xe^{x/3}. \]
   What is the unique solution satisfying \( y(0) = 2 \)?

4. Find the most general solution to the ODE:
   \[ xe^y \frac{dy}{dx} + \frac{x^2 + 1}{y} = 0, \quad x > 0. \]

5. A tank contains 200 l of a dye solution with a concentration of 1 g/l. The tank is rinsed with fresh water flowing in at a rate of 2 l/min. A well-stirred solution flows out at the same rate. How long does it take for the dye concentration in the tank to reach 1% of its original value?

6. Find the unique solution to the ODE:
   \[ y' + y = 5 \sin 2t, \]
   with initial condition \( y(0) = 1 \).

7. Find the unique solution to the ODE:
   \[ 2y' + y = 3t^2, \]
   with \( y(1) = 0 \).

8. The population of a certain mammal \( P(t) \) satisfies the ODE:
   \[ \frac{dP}{dt} = r \left( 1 - \frac{P}{K} \right) P, \]
   for \( t \geq 0 \). The constants \( r > 0 \) and \( K > 0 \) represent the growth rate and the saturation level, respectively. Find the population \( P(t) \) if the initial population \( P(0) \) satisfies \( 0 < P(0) < K \), and if it satisfies \( P(0) > K \).

9. Find a general formula for the Wronskian of two solutions to the ODE:
   \[ 2t^2 y'' + 4ty' - y = 0. \]
   When can we be sure that the two solutions are independent?
10. Find the unique solution to the initial-value problem:

\[ y'' + 4y' + 5y = 0, \]

with \( y(0) = 1 \), and \( y'(0) = 0 \).

11. Consider the following nonhomogeneous, second-order ODE:

\[ y''(t) - y'(t) - 6y(t) = 2e^{3t}. \]

Find a set of independent solutions to the associated homogeneous ODE. Find the unique solution to the nonhomogeneous ODE with \( y(0) = 0 \) and \( y'(0) = 1 \).

12. Find a set of independent solutions for the ODE:

\[ y'' - 6y' + 9y = 0. \]

13. Find a particular solution to the ODE:

\[ 4y'' - 4y' + y = 16e^{t/2}. \]

14. Find the unique solution to the first-order initial-value problem:

\[ \sqrt{2xy} \frac{dy}{dx} = 1, \quad x > 0, \]

and the condition \( y(0) = 1 \).

15. a. Sketch the function \( h(t) \) and write it, for \( t \geq 0 \), in terms of the step functions \( u_c(t) \):

\[
h(t) = \begin{cases} 
0 & 0 \leq t < 1 \\
t - 1 & 1 \leq t < 2 \\
1 & t \geq 2.
\end{cases}
\]

b. Find the Laplace transform of the function in (a.).

c. Find the inverse Laplace transform of the function

\[ F(s) = \frac{e^{-4s} + e^{-2s}}{s^2 + 2s + 1}. \]

16. Find the unique solution to the ODE:

\[ y'' + y = 0, \]

with \( y(\pi/3) = 2 \), and \( y'(\pi/3) = -4 \).

17. Find the unique solution to the initial value problem:

\[ y''(t) + 2y(t) = \begin{cases} 
\cos t & 0 \leq t < \pi \\
0 & t \geq \pi,
\end{cases} \]

with \( y(0) = 0 = y'(0) \). Note that \( \cos(t - \pi) = -\cos t \).

18. Find the inverse Laplace transform of the function

\[ F(s) = \frac{e^{-2s}}{s^2 + 2s + 3} + 2e^{-4s}. \]

19. Find the unique solution to the ODE:

\[ y''(t) + 6y(t) = 5\delta(t - 4), \]

with initial conditions \( y(0) = 1 \) and \( y'(0) = 0 \).