MA214 Section 003 Fall 2009 Review for the Final Exam

FINAL EXAM: 16 December 2009, 10:30-12:30, CB 203

1. Find the most general solution to the ODE:

$$4y'(t) + y(t) = 8t.$$

2. Find the most general solution to the ODE:

$$x\frac{dy}{dx} + y = \sin x.$$

3. Find the most general solution to the ODE:

$$3\frac{dy}{dx} - y = xe^{x/3}.$$

What is the unique solution satisfying y(0) = 2?

4. Find the most general solution to the ODE:

$$xe^{y}\frac{dy}{dx} + \frac{x^{2}+1}{y} = 0, \quad x > 0.$$

- 5. A tank contains 200 l of a dye solution with a concentration of 1 g/l. The tank is rinsed with fresh water flowing in at a rate of 2 l/min. A well-stirred solution flows out at the same rate. How long does it take for the dye concentration in the tank to reach 1% of its original value?
- 6. Find the unique solution to the ODE:

$$y' + y = 5\sin 2t,$$

with initial condition y(0) = 1.

7. Find the unique solution to the ODE:

$$2y' + y = 3t^2,$$

with y(1) = 0.

8. The population of a certain mammal P(t) satisfies the ODE:

$$\frac{dP}{dt} = r\left(1 - \frac{P}{K}\right)P,$$

for $t \ge 0$. The constants r > 0 and K > 0 represent the growth rate and the saturation level, respectively. Find the population P(t) if the initial population P(0) satisfies 0 < P(0) < K, and if it satisfies P(0) > K.

9. Find a general formula for the Wronskian of two solutions to the ODE:

$$2t^2y'' + 4ty' - y = 0.$$

When can we be sure that the two solutions are independent?

10. Find the unique solution to the initial-value problem:

$$y'' + 4y' + 5y = 0,$$

with y(0) = 1, and y'(0) = 0.

11. Consider the following nonhomogeneous, second-order ODE:

$$y''(t) - y'(t) - 6y(t) = 2e^{3t}.$$

Find a set of independent solutions to the associated homogeneous ODE. Find the unique solution to the nonhomogeneous ODE with y(0) = 0 and y'(0) = 1.

12. Find a set of independent solutions for the ODE:

y'' - 6y' + 9y = 0.

13. Find a particular solution to the ODE:

$$4y'' - 4y' + y = 16e^{t/2}.$$

14. Find the unique solution to the first-order initial-value problem:

$$\sqrt{2xy}\frac{dy}{dx} = 1, \quad x > 0,$$

and the condition y(0) = 1.

15. a. Sketch the function h(t) and write it, for $t \ge 0$, in terms of the step functions $u_c(t)$:

$$h(t) = \begin{cases} 0 & 0 \le t < 1\\ t - 1 & 1 \le t < 2\\ 1 & t \ge 2. \end{cases}$$

- b. Find the Laplace transform of the function in (a.).
- c. Find the inverse Laplace transform of the function

$$F(s) = \frac{e^{-4s} + e^{-2s}}{s^2 + 2s + 1}.$$

16. Find the unique solution to the ODE:

$$y'' + y = 0,$$

with $y(\pi/3) = 2$, and $y'(\pi/3) = -4$.

17. Find the unique solution to the initial value problem:

$$y''(t) + 2y(t) = \begin{cases} \cos t & 0 \le t < \pi \\ 0 & t \ge \pi, \end{cases}$$

with y(0) = 0 = y'(0). Note that $\cos(t - \pi) = -\cos t$.

18. Find the inverse Laplace transform of the function

$$F(s) = \frac{e^{-2s}}{s^2 + 2s + 3} + 2e^{-4s}.$$

19. Find the unique solution to the ODE:

$$y''(t) + 6y(t) = 5\delta(t - 4),$$

with initial conditions y(0) = 1 and y'(0) = 0.