Quiz #1 - 10 Points
11 September 2009

NAME: Solutions

1. (4 points). Find the most general solution to

$$x^2 \frac{dy}{dx} = y + 1.$$

Separate variables: $\frac{dy}{y+1} = \frac{1}{x^2}dx$

Integrate: $\ln(y+1) = -\frac{1}{x} + C$

Solve: $y(x) + 1 = Ce^{-\frac{1}{x}}$

$\boxed{y(x) = Ce^{-\frac{1}{x}} - 1}$

Check: $y' = (Ce^{-\frac{1}{x}})(\frac{1}{x^2})$ so $x^2 y' = (Ce^{-\frac{1}{x}} - 1) + 1 = y + 1$ ✓

2. (3 points). Find the unique solution to

$$2y' + y = e^{-x/2}, \quad y(0) = 0.$$

$y' + \frac{1}{2}y = \frac{1}{2}e^{-x/2}$

$p = \frac{1}{2} \quad q = \frac{1}{2}e^{-x/2}$

General Soln: $\mu = e^{\int p\,dx} = e^{x/2}$

$(\mu y)' = \mu q = e^{x/2} \cdot \frac{1}{2}e^{-x/2} = \frac{1}{2}$

$\mu y = \frac{1}{2}x + C$ or $\boxed{y(x) = \frac{1}{2}xe^{-x/2} + Ce^{-x/2}}$

Initial Cond. $y(0) = C = 0$ so

$\boxed{y(x) = \frac{1}{2}xe^{-x/2}}$

2. (3 points). ON THE BACK: Sketch the direction field of the ODE $y' = 1 + y$. For any solution curve passing through the point (x_0, y_0) with $y_0 > 0$, what is $\lim_{t\to+\infty} y(t)$?

$f(x,y) = 1 + y$ indep. of x.

if $y_0 > 0$ then

$\lim y(x) = +\infty$

if $y_0 < 0$

$\lim y(x) = -\infty$.