

NAME: \_\_\_\_\_

Solutions

1. (4 points). Find the most general solution to

$$x^2 \frac{dy}{dx} = y + 1.$$

Separate variables:  $\frac{dy}{y+1} = \frac{1}{x^2} dx$

Integrate:  $\ln(y+1) = -\frac{1}{x} + C$

Solve:  $y(x)+1 = Ce^{-\frac{1}{x}}$

$$y(x) = Ce^{-\frac{1}{x}} - 1$$

Check:  $y' = (Ce^{-\frac{1}{x}}) \left(\frac{1}{x^2}\right)$  so  $x^2 y' = (Ce^{-\frac{1}{x}} - 1) + 1 = y + 1$  ✓

2. (3 points). Find the unique solution to

$$2y' + y = e^{-x/2}, \quad y(0) = 0.$$

$$y' + \frac{1}{2}y = \frac{1}{2}e^{-x/2}$$

$$p = \frac{1}{2} \quad q = \frac{1}{2}e^{-x/2}$$

General Soln:  $\mu = e^{\int p dx} = e^{x/2}$

$$(\mu y)' = \mu q = e^{x/2} \cdot \frac{1}{2}e^{-x/2} = \frac{1}{2}$$

$$\mu y = \frac{1}{2}x + C \quad \text{or} \quad y(x) = \frac{1}{2}xe^{-x/2} + Ce^{-x/2}$$

Initial Cond.  $y(0) = C = 0$  so

$$y(x) = \frac{1}{2}xe^{-x/2}$$

2. (3 points). ON THE BACK: Sketch the direction field of the ODE  $y' = 1 + y$ .  
For any solution curve passing through the point  $(x_0, y_0)$  with  $y_0 > 0$ , what is  $\lim_{t \rightarrow +\infty} y(t)$ ?

$$f(x, y) = 1 + y \quad \text{indep. of } x.$$

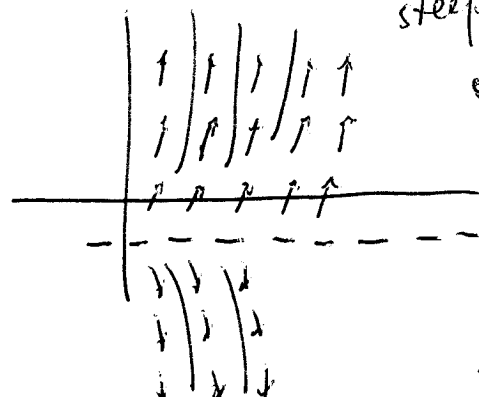
if  $y_0 > 0$  then

$$\lim_{t \rightarrow \infty} y(t) = +\infty$$

if  $y_0 < 0$

$$\lim_{t \rightarrow \infty} y(t) = -\infty$$

1



steeper

•  $y > -1$   
positive slope

•  $y = -1$   
zero slope

•  $y < -1$  neg.