MA214–03 Fall 2009
Quiz #2 - 10 Points
25 September 2009

NAME: Solutions

1. (4 points). A body falls from the top of a 490 meter tower in the earth's gravitational field so $F = -mg$ with $g = 9.8m/s^2$. How long does it take to reach the ground?

$m\ddot{x} = -mg \Rightarrow \frac{d^2x}{dt^2} = -g \quad \frac{dx}{dt} = -gt + C \quad x(t) = -\frac{1}{2}gt^2 + Ct + D$

$x(t=0) = 490m$ and $\dot{x}(t=0) = 0$ $\Rightarrow x(t) = -\frac{1}{2}gt^2 + 0 \cdot t + 490 = -\frac{1}{2}gt^2 + 490$

When it hits the ground, $x = 0$ so $\frac{1}{2}gt^2 = 490$

$t^2 = 100$ so $\boxed{t = 10s}$

2. (6 points). A population of squirrels on campus begins with an initial population of 10. The population increases at a rate of 10% per year. The campus hawks eat 5 squirrels per year. Assume the growth and eating are done continuously throughout the year. How long does it take for the squirrel population to go to zero? The ODE satisfied by the population $P(t)$ at time $t \geq 0$ is

$$P'(t) = rP(t) + k.$$

$r = 0.1$

$k = -5$ negative

$P'(t) = \frac{1}{10}P - 5$

$10 \log\left(\frac{1}{10}P - 5\right) = t + C$

$\frac{1}{10}P - 5 = Ce^{\frac{1}{10}t}$

$\boxed{P(t) = Ce^{\frac{1}{10}t} + 50}$

$P(t=0) = 10 = C + 50$

$C = -40$

$\boxed{P(t) = -40e^{\frac{1}{10}t} + 50}$

$P(T) = 0$ means $40e^{\frac{1}{10}T} = 50$

$e^{\frac{1}{10}T} = \frac{5}{4}$ or $\boxed{T = 10 \ln 5/4}$

a bit more than 2.2 years.