MA214–03 Fall 2009
Quiz #2 - 10 Points
16 October 2009

NAME: Solutions

1. (5 points). Find the most general solution to the second-order ODE:

$$y'' - 6y' + 9y = 0.$$

Characteristic eqn: $r^2 - 6r + 9 = (r-3)^2 = 0$

$r = 3$ double root

2 independent solns: $y_1(t) = e^{3t}$ $y_2(t) = te^{3t}$

General Soln: $\boxed{y(t) = C_1 e^{3t} + C_2 te^{3t}}$

2. (5 points). Find the unique solution to the second-order ODE:

$$y'' + y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

Characteristic eqn: $r^2 + r - 6 = (r+3)(r-2) = 0$

2 distinct real roots $r_1 = -3$ & $r_2 = 2$

2 independent solns: $y_1(t) = e^{-3t}$ $y_2(t) = e^{2t}$

General soln: $y(t) = C_1 e^{-3t} + C_2 e^{2t}$

$y'(t) = -3C_1 e^{-3t} + 2C_2 e^{2t}$

Initial Conditions: $y(0) = C_1 + C_2 = 1$

$y'(0) = -3C_1 + 2C_2 = 0 \Leftrightarrow C_2 = \frac{3}{2}C_1$

so $C_1 + C_2 = \frac{5}{2}C_1 = 1$ $C_1 = \frac{2}{5}$ $C_2 = \frac{3}{5}$

Unique Soln: $\boxed{y(t) = \frac{2}{5}e^{-3t} + \frac{3}{5}e^{2t}}$