MA214–03 Fall 2009
Quiz #4 - 10 Points
2 November 2009

NAME: Solutions

1. (5 points). Find the most general **real** solution to the second-order ODE:

$$y'' + 4y' + 5y = 0.$$

Characteristic eqn: $r^2 + 4r + 5 = 0$ $\quad b^2 - 4ac = 16 - 20 = -4 < 0$

2 complex roots: $\dfrac{-4 \pm [-4]^{\frac{1}{2}}}{2} = -2 \pm i$

2 complex solns: $e^{(-2 \pm i)t} = e^{-2t} e^{\pm it} = e^{-2t}(\cos t \pm i \sin t)$

2 Indep. real solns: $y_1(t) = e^{-2t}\cos t \quad y_2(t) = e^{-2t}\sin t$

General real soln: $\boxed{y(t) = C_1 e^{-2t}\cos t + C_2 e^{-2t}\sin t}$

2. (5 points). Find the most general **real** solution of the nonhomogeneous ODE:

$$y'' + 4y' + 5y = e^{-t}.$$

Intelligent guess: $y_p(t) = Ae^{-t}$

since $g(t) = e^{-t}$ is independent of y_1 & y_2.

$y_p'(t) = -Ae^{-t} \quad y_p''(t) = Ae^{-t}$

substitute: $(A - 4A + 5A)e^{-t} = e^{-t} \iff A = \frac{1}{2}$

$\boxed{y_p(t) = \frac{1}{2}e^{-t}}$

Most general soln: $\boxed{y(t) = C_1 e^{-2t}\cos t + C_2 e^{-2t}\sin t + \frac{1}{2}e^{-t}}$