NAME: Solutions

1. (5 points). Find the most general real solution to the second-order ODE:

\[ y'' + 4y' + 5y = 0. \]

Characteristic eqn: \[ r^2 + 4r + 5 = 0 \]
\[ b^2 - 4ac = 16 - 20 = -4 < 0 \]
2 complex roots: \[ r = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i \]
2 complex solns: \[ e^{(-2\pm i)t} = e^{-2t}e^{\pm it} = e^{-2t} (\cos t \pm i \sin t) \]
2 independent real solns: \[ y_1(t) = e^{-2t} \cos t \quad y_2(t) = e^{-2t} \sin t \]
General real soln: \[ y(t) = C_1 e^{-2t} \cos t + C_2 e^{-2t} \sin t \]

2. (5 points). Find the most general real solution of the nonhomogeneous ODE:

\[ y'' + 4y' + 5y = e^{-t}. \]

Intelligent guess: \[ y_p(t) = Ae^{-t} \]

since \( g(t) = e^{-t} \) is independent of \( y_1 \) and \( y_2 \):

\[ y_p'(t) = -Ae^{-t} \quad y_p''(t) = Ae^{-t} \]

Substitute: \( (A - 4A + 5A)e^{-t} = e^{-t} \iff A = \frac{1}{2} \]

\[ y_p(t) = \frac{1}{2} e^{-t} \]

Most general soln: \[ y(t) = C_1 e^{-2t} \cos t + C_2 e^{-2t} \sin t + \frac{1}{2} e^{-t} \]