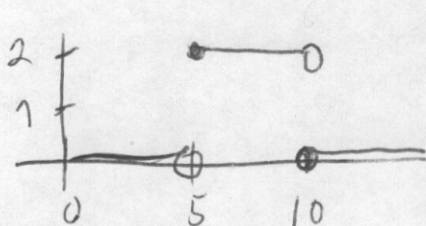


NAME: Solutions

1. (4 points). Find the Laplace transform of the square pulse:

$$g(t) = \begin{cases} 0 & 0 \leq t < 5 \\ 2 & 5 \leq t < 10 \\ 0 & 10 \leq t. \end{cases}$$

First graph the function $g(t)$ and write $g(t)$ in terms of the step functions $u_c(t)$.



$$g(t) = 2(u_5(t) - u_{10}(t))$$

$$(Lg)(s) = \frac{2}{s} (e^{-5s} - e^{-10s})$$

2. (6 points). Find the inverse Laplace transform of the function:

$$F(s) = \frac{e^{-2s}}{s^2 + 4} + \frac{1}{s^2 + 2s + 10}$$

$$\textcircled{1} \quad \frac{e^{-2s}}{s^2 + 4} = e^{-2s} H(s) \rightarrow u_2(t) (\mathcal{L}^{-1} H)(t-2) = \frac{1}{2} u_2(t) \sin(2(t-2))$$

$$\text{ILT} \quad (\mathcal{L}^{-1} H)(t) = \frac{1}{2} \sin 2t$$

$$\textcircled{2} \quad \frac{1}{(s+1)^2 + 9} = \frac{1}{3} \frac{3}{(s+1)^2 + 3^2} \rightarrow \frac{1}{3} e^{-t} \sin(3t)$$

$$(\mathcal{L}^{-1} F)(t) = \frac{1}{2} u_2(t) \sin(2(t-2)) + \frac{1}{3} e^{-t} \sin(3t)$$