MA214 Fall 2009
Practice Test #2: Test #2 on Friday, 20 November 2009

REVIEW SESSION: Wednesday, 18 November, 4PM–5PM, CB 313

MATERIAL: All of Chapter 3 and section 6.1.

1. Show that $y_1(x) = x$ and $y_2(x) = xe^x$ form an independent set of solutions to the ODE:

$$x^2y'' - x(x+2)y' + (x+2)y = 0, \quad x > 0.$$

2. Are the two functions $w_1(t) = t(t+1)$ and $w_2(t) = t^2$ independent on the real line?

3. Find the most general solution to the ODE:

$$y''(t) - ty'(t) = t,$$

using the substitution $u(t) = y'(t)$.

4. Find the unique solution to the initial value problem

$$y'' + 5y' - 6y = 0,$$

with the conditions $y(0) = 1$ and $y'(0) = 0$.

5. Compute the Laplace transform of $f(t) = e^{2t}t^2$ and of $g(t) = t\sin(4t)$.

6. Use Abel's formula to compute the Wronskian of two solutions to the ODE

$$2t^2y'' + 3ty' - y = 0, \quad t > 0.$$

7. Use variation of parameters to find a particular solution to

$$y'' + 4y' + 4y = t^{-2}e^{-2t}.$$

8. Find the Laplace transform of the solution of the initial value problem:

$$2y''(t) - 3y'(t) + y(t) = 2\sin(2t), \quad y(0) = 1, y'(0) = 1.$$

9. Find the most general real solution to the ODE:

$$y'' + 2y' + 2y = 4t.$$

10. Find the unique solution to the initial value problem

$$4y'' + 12y' + 9y = 0,$$

with $y(0) = 1$, and $y'(0) = 1$.

11. Find the unique solution to the initial value problem

$$y'' + 4y = 3\sin 2t,$$

with $y(0) = 2$ and $y'(0) = -1$.

12. Consider a driven, undamped harmonic oscillator described by the ODE

$$u'' + 2u = 2\cos\omega t.$$

What is the natural frequency? Find the solutions for ω not equal to, and for ω equal to, the natural frequency, when the initial conditions are $u(0) = 0$ and $u'(0) = 0$.

13. What is a set of independent *real* solutions for the damped, undriven oscillator described by

$$u'' + 4u' + 4u = 0.$$

What is the unique solution to this ODE with initial conditions $u(0) = 1$ and $u'(0) = 0$? How long does it take for the amplitude to decrease to one-half of its initial value?

14. Use the method of undetermined coefficients to find the unique solution of

$$y'' + 4y = t, \quad y(0) = 1, y'(0) = 0.$$