Quiz #1 - 10 Points
29 January 2010

NAME: Solutions

1. (5 points). Find the most general solution to

$$\frac{dy}{dt} + 2ty(t) = 2t.$$

2 methods

1) Integrating factor: $\mu = e^{\int 2t\,dt} = e^{t^2}$ so $(\mu y)' = \mu g = 2te^{t^2}$

$\Rightarrow \mu y = \int 2te^{t^2}dt = \int e^{z}dz = e^{t^2} + C$ so $\boxed{y(t) = 1 + Ce^{-t^2}}$

let $z = t^2$, $dz = 2t\,dt$

Check: $y'(t) = -2tCe^{-t^2}$ note $Ce^{-t^2} = y - 1$ so $y' = -2t(y-1) = -2ty + 2t$ ✓

2) Separate: $y' = 2t(1-y)$ or $\int \frac{dy}{1-y} = \int 2t\,dt \Rightarrow \ln|1-y| = -t^2 + c$

so $1 - y = Ce^{-t^2} \Rightarrow \boxed{y(t) = 1 + Ce^{-t^2}}$

2. (5 points). You are given the ODE: $y'(t) = 2 - y(t)$

a. What is the direction field? Sketch the direction field mentioning any important properties.

b. Solve the ODE

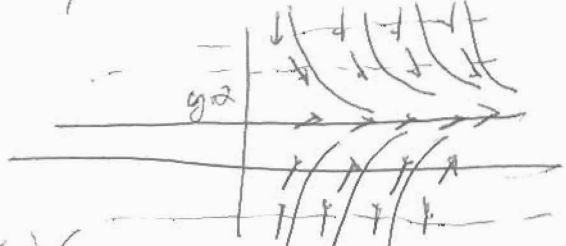
c. What is the limit of $y(t)$ as $t \to \infty$?

a) $f(t,y) = 2 - y$ is the direction field. It is independent of t

It vanishes at $y = 2$. This is the equilibrium soln.

$y > 2$ $f < 0$ so $y' < 0$

$y < 2$ $f > 0$ so $y' > 0$



b.) $\int \frac{dy}{2-y} = \int dt \Rightarrow \ln|2-y| = -t + C$

$2 - y = Ce^{-t}$ or $\boxed{y(t) = 2 + Ce^{-t}}$

c.) $\lim_{t\to\infty} y(t) = 2$.

Check: $y' = -Ce^{-t}$. $Ce^{-t} = y - 2$ so

$y' = -(y-2) = -y + 2$ ✓.