MA214-03 Spring 2010
Quiz #2 - 10 Points
12 February 2010

NAME: Solutions

1. (4 points). A body falls from the top of a 490 meter tower in the earth's gravitational field so $F = -mg$ with $g = 9.8m/s^2$. How long does it take to reach the ground?

$mx'' = -mg$ so $x''(t) = -g$

Integrate: $x'(t) = -gt + v_0$

$\boxed{x(t) = -\frac{1}{2}gt^2 + v_0 t + x_0}$

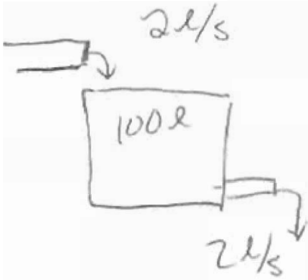
Initial Cond. $x(t=0) = 490\ m$ $\quad v(t=0) = 0$

$x(t) = -\frac{1}{2}gt^2 + 490 = -4.9t^2 + 490$

On the ground, $x(T) = 0 = -4.9T^2 + 490 \Rightarrow T^2 = 100\ \text{sec} \Rightarrow$ $\boxed{T = 10\ \text{sec}}$

2. (6 points). A 100 liter tank is initially filled with pure water. At time zero, polluted water with 2 grams per liter of radium pours into the tank at a rate of 2 liters per second. The tank is drained at the same rate.

1. Find the formula for the amount of radium in the tank at any time.
2. Estimate (no calculators) how many grams of radium are in the tank after 50 seconds.



$Q(t) \equiv$ amount of radium in tank at time t in grams

$Q(t=0) = 0$ (pure water)

$$\frac{dQ}{dt} = \frac{2g}{\ell} \cdot \frac{2\ell}{sec} - \frac{Q(t)}{100}\frac{g}{\ell} \cdot \frac{2\ell}{s}$$

$$\boxed{\frac{dQ}{dt} = 4 - \frac{Q(t)}{50}}$$

$$\int \frac{dQ}{4 - \frac{Q}{50}} = -50\int \frac{du}{u} = -50 \ln\left|4 - \frac{Q}{50}\right| = t + C$$

$u = 4 - \frac{1}{50}Q$

$du = -\frac{1}{50}dQ$

$\ln\left|4 - \frac{Q}{50}\right| = -t/50 + C$

$4 - \frac{Q}{50} = Ce^{-t/50}$

$Q(t) = 200 + Ce^{-t/50}$ General Soln.

Initial Cond: $Q(0) = 0$ $\quad C = -200$

a) $\boxed{Q(t) = 200\left(1 - e^{-t/50}\right)}$

b) $Q(50) = 200(1 - e$

$\approx 200(1 - \frac{1}{2}$

$\boxed{Q(50) \approx 100\ gm}$