1. (7 points). Consider the ODE:

\[ y'' - 2y' + y = 0. \]

a. Find the most general solution to the second-order ODE.

b. Find the unique solution satisfying the initial conditions:

\[ y(0) = 1, \quad y'(0) = 0. \]

Characteristic eqn: \[ r^2 - 2r + 1 = 0 \]

- \( \frac{-b}{2a} = 1 \) so:

\[ \Delta > 0 \text{ double root} \]

a) General soln:

\[ y_1(t) = e^t \quad \text{&} \quad y_2(t) = te^t \]

2 independent solns

b) Unique soln with

\[ y(0) = 1 = C_1 + C_2 \]

\[ y'(0) = 0 = C_1 + C_2 \]

\[ y(0) = 1 = C_1 + C_2 \]

\[ y(0) = 1 = C_1 + C_2 \]

\[ y(t) = e^t - te^t = (1-t)e^t \]

2. (3 points). Complex algebra:

a. Compute the product: \((2 - 3i)(4 + i)\)

b. Find \(|z|\), when \(z = 2 + 3i\)

c. If \(z = 1 - i\), find the complex conjugate \(\bar{z}\).

\[ (2 - 3i)(4 + i) = 8 - 12i + 2i + 3(-i)(i) = 11 - 10i \]

b. \[ |z| = \sqrt{2^2 + 3^2} = \sqrt{13} \]

c. \[ \bar{z} = 1 + i \]