

Solutions

NAME: _____

1. (7 points). Consider the ODE:

$$y'' - 2y' + y = 0.$$

- a. Find the most general solution to the second-order ODE.
 b. Find the unique solution satisfying the initial conditions:

$$y(0) = 1, \quad y'(0) = 0.$$

Characteristic eqn: $r^2 - 2r + 1 = 0$ $b^2 - 4ac = (-2)^2 - 4 = 0$ double root
 $-b/2a = 1$ so: $y_1(t) = e^t$ & $y_2(t) = te^t$ 2 independent solns

a) general soln:

$$y(t) = C_1 e^t + C_2 t e^t$$

$$y'(t) = C_1 e^t + C_2 e^t + C_2 t e^t$$

b) Unique soln. with $y(0) = 1 = C_1$ } $C_1 = 1$ & $C_2 = -1$
 $y'(0) = 0 = C_1 + C_2$ }

$$y(t) = e^t - t e^t = (1-t)e^t$$

2. (3 points). Complex algebra:

- a. Compute the product: $(2 - 3i)(4 + i)$
 b. Find $|z|$, when $z = 2 + 3i$
 c. If $z = 1 - i$, find the complex conjugate \bar{z} .

a. $(2 - 3i)(4 + i) = 8 - 12i + 2i + 3(-i)(i) = \boxed{11 - 10i}$

b. $|z| = \sqrt{2^2 + 3^2} = \sqrt{13}$

c. $\bar{z} = 1 + i$