MA214–03 Spring 2010
Quiz #4 - 10 Points
26 March 2010

NAME: Solutions

1. (10 points). Consider the ODE:

$$y'' + 3y' + 2y = e^{-t}.$$

a. Find the most general solution to this ODE.

b. Find the unique solution to the initial-value problem for this ODE:

$$y(0) = 1, \quad y'(0) = 0.$$

1) homog. ODE: $y'' + 3y' + 2y = 0 \Rightarrow r^2 + 3r + 2 = (r+2)(r+1) = 0$

2 roots: -2 & -1. 2 indep. solns. $\boxed{y_1(t) = e^{-t} \ \& \ y_2(t) = e^{-2t}}$

$\boxed{y_h(t) = C_1 e^{-t} + C_2 e^{-2t}}$.

$$W(y_1, y_2)(t) = \begin{vmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{vmatrix} = -2e^{-3t} - (-e^{-3t}) = \boxed{-e^{-3t} = W(t)}$$

2 Particular soln: Since $g(t) = y_1(t)$ it is not independent so use Variation of Parameters:

$$u_1'(t) = \frac{-y_2(t) g(t)}{W(t)} = -\frac{e^{-2t} \cdot e^{-t}}{-e^{-3t}} = 1 \Rightarrow u_1(t) = t$$

$$u_2'(t) = \frac{+y_1(t) g(t)}{W(t)} = \frac{e^{-t} e^{-t}}{-e^{-3t}} = -e^{t} \Rightarrow u_2(t) = -e^{t}$$

$y_p(t) = u_1(t) y_1(t) + u_2(t) y_2(t) = te^{-t} - e^{-t}$. Since $-e^{-t}$ solves the homog ODE we can take $\boxed{y_p(t) = te^{-t}}$

a) General Soln: $y_g(t) = y_h(t) + y_p(t) = \boxed{C_1 e^{-t} + C_2 e^{-2t} + te^{-t} = y_g(t)}$

$y_g'(t) = -C_1 e^{-t} - 2C_2 e^{-2t} + e^{-t} - te^{-t}$

b) $\left.\begin{aligned} y_g(0) &= C_1 + C_2 = 1 \\ y_g'(0) &= -C_1 - 2C_2 + 1 = 0 \end{aligned}\right\}$ add $\Rightarrow -C_2 + 1 = 1 \Rightarrow C_2 = 0$

$C_1 = 1$

$$\boxed{y(t) = e^{-t}(1 + t)}$$