MA214–03 Spring 2010
Quiz #5 - 10 Points
16 April 2010

NAME: Solutions

1. Use the Laplace transform method to find the unique solution ODE:

$$y''(t) + 2y(t) = 1,$$

with initial conditions $y(0) = 0$ and $y'(0) = 1$. Formulas are on the board.

Step 1

$$\mathcal{L}(y'' + 2y)(s) = (\mathcal{L}1)(s) = \frac{1}{s}$$

$$s^2(\mathcal{L}y)(s) - y'(0) - sy(0) + 2(\mathcal{L}y)(s) = \frac{1}{s}$$

$$(s^2+2)(\mathcal{L}y)(s) - 1 = \frac{1}{s}$$

$$\boxed{(\mathcal{L}y)(s) = \frac{1}{s(s^2+2)} + \frac{1}{s^2+2}}$$

Step 2 ILT

$$\frac{1}{s(s^2+2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2}$$

$$1 = A(s^2+2) + (Bs+C)s = (A+B)s^2 + Cs + 2A$$

$$C = 0 \quad A = -B \quad 2A = 1 \quad A = \frac{1}{2} \quad B = -\frac{1}{2}.$$

$$(\mathcal{L}y)(s) = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{s}{s^2+2} + \frac{1}{s^2+2}$$

$$\downarrow$$

$$\boxed{y(t) = \frac{1}{2} - \frac{1}{2}\cos(\sqrt{2}t) + \frac{1}{\sqrt{2}}\sin(\sqrt{2}t)}$$

Check: $y(0) = \frac{1}{2} - \frac{1}{2} = 0$ ✓ 1

$y'(t) = \frac{\sqrt{2}}{2}\sin\sqrt{2}t + \cos\sqrt{2}t \Rightarrow y'(0) = 1$ ✓