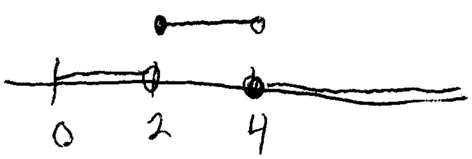
MA214–03 Spring 2010
Quiz #6 - 10 Points
23 April 2010

NAME: Solutions

1. (4 points). Find the Laplace transform of the square pulse:

$$g(t) = \begin{cases} 0 & 0 \le t < 2 \\ 2 & 2 \le t < 4 \\ 0 & 4 \le t. \end{cases}$$

First graph the function $g(t)$ and write $g(t)$ in terms of the step functions $u_c(t)$.



$$g(t) = 2u_2(t) - 2u_4(t)$$

$$\boxed{(\mathcal{L}g)(s) = \frac{2e^{-2s}}{s} - \frac{2e^{-4s}}{s}}$$

2. (6 points). Find the inverse Laplace transform of the functions:

$$F(s) = \frac{e^{-3s}}{s^2+4},$$

and

$$G(s) = \frac{s}{s^2+2s+10}.$$

$$F(s) = \frac{1}{2}e^{-3s}\left(\frac{2}{s^2+2^2}\right) \rightarrow \boxed{\frac{1}{2}u_3(t)\sin 2(t-3)}$$

$G(s)$: $s^2+2s+10 = (s+1)^2+3^2$ (irreducible)

$$\frac{s}{s^2+2s+10} = \frac{s}{(s+1)^2+3^2} = \frac{(s+1)}{(s+1)^2+3^2} - \frac{1}{(s+1)^2+3^2}$$

① ILT: $e^{-t}\cos 3t$

② ILT: $-\frac{1}{3}\frac{3}{(s+1)^2+3^2} \rightarrow -\frac{1}{3}e^{-t}\sin 3t$

Result: $\boxed{(\mathcal{L}^{-1}G)(t) = e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t}$