MA214 Section 003 Spring 2010
Review for the Final Exam

FINAL EXAM: Wednesday, 5 May 2010, 10:30–12:30, CB 339

REVIEW FOR THE FINAL: Tuesday, 4 May 2010 4PM in CB 219.

Material: Chapter 1, Chapter 2 (except sections 2.4, 2.6, 2.8, and 2.9), Chapter 3, and Chapter 6 (sections 6.1-6.5).

1. Find the most general solution to the ODE:
$$4y'(t) + y(t) = 8t.$$
2. Find the most general solution to the ODE:
$$x\frac{dy}{dx} + y = \sin x.$$
3. Find the most general solution to the ODE:
$$3\frac{dy}{dx} - y = xe^{x/3}.$$
What is the unique solution satisfying $y(0) = 2$?
4. Find the most general solution to the ODE:
$$xe^y\frac{dy}{dx} + \frac{x^2+1}{y} = 0, \quad x > 0.$$
5. A tank contains 200 l of a dye solution with a concentration of 1 g/l. The tank is rinsed with fresh water flowing in at a rate of 2 l/min. A well-stirred solution flows out at the same rate. How long does it take for the dye concentration in the tank to reach 1% of its original value?
6. Find the unique solution to the ODE:
$$y' + y = 5\sin 2t,$$
with initial condition $y(0) = 1$.
7. Find the unique solution to the ODE:
$$2y' + y = 3t^2,$$
with $y(1) = 0$.
8. The population of a certain mammal $P(t)$ satisfies the ODE:
$$\frac{dP}{dt} = r\left(1 - \frac{P}{K}\right)P,$$
for $t \geq 0$. The constants $r > 0$ and $K > 0$ represent the growth rate and the saturation level, respectively. Find the population $P(t)$ if the initial population $P(0)$ satisfies $0 < P(0) < K$, and if it satisfies $P(0) > K$.
9. Find a general formula for the Wronskian of two solutions to the ODE:
$$2t^2y'' + 4ty' - y = 0.$$
When can we be sure that the two solutions are independent?

10. Find the unique solution to the initial-value problem:

$$y'' + 4y' + 5y = 0,$$

with $y(0) = 1$, and $y'(0) = 0$.

11. Consider the following nonhomogeneous, second-order ODE:

$$y''(t) - y'(t) - 6y(t) = 2e^{3t}.$$

Find a set of independent solutions to the associated homogeneous ODE. Find the unique solution to the nonhomogeneous ODE with $y(0) = 0$ and $y'(0) = 1$.

12. Find a set of independent solutions for the ODE:

$$y'' - 6y' + 9y = 0.$$

13. Find a particular solution to the ODE:

$$4y'' - 4y' + y = 16e^{t/2}.$$

14. Find the unique solution to the first-order initial-value problem:

$$\sqrt{2xy}\frac{dy}{dx} = 1, \quad x > 0,$$

and the condition $y(0) = 1$.

15. a. Sketch the function $h(t)$ and write it, for $t \geq 0$, in terms of the step functions $u_c(t)$:

$$h(t) = \begin{cases} 0 & 0 \leq t < 1 \\ t - 1 & 1 \leq t < 2 \\ 1 & t \geq 2. \end{cases}$$

b. Find the Laplace transform of the function in (a.).

c. Find the inverse Laplace transform of the function

$$F(s) = \frac{e^{-4s} + e^{-2s}}{s^2 + 2s + 1}.$$

16. Find the unique solution to the ODE:

$$y'' + y = 0,$$

with $y(\pi/3) = 2$, and $y'(\pi/3) = -4$.

17. Find the unique solution to the initial value problem:

$$y''(t) + 2y(t) = \begin{cases} \cos t & 0 \leq t < \pi \\ 0 & t \geq \pi, \end{cases}$$

with $y(0) = 0 = y'(0)$. Note that $\cos(t - \pi) = -\cos t$.

18. Find the inverse Laplace transform of the function

$$F(s) = \frac{e^{-2s}}{s^2 + 2s + 3} + 2e^{-4s}.$$

19. Find the unique solution to the ODE:

$$y''(t) + 6y(t) = 5\delta(t - 4),$$

with initial conditions $y(0) = 1$ and $y'(0) = 0$.