Quiz #1 - 10 Points
28 January 2011

NAME: Solutions

1. (5 points). Find the most general solution to the ODE $\frac{dy}{dx} = xy$.

Separate $\int \frac{dy}{y} = \int x\,dx$

$\ln|y| = \frac{1}{2}x^2 + C$

$y(x) = Ce^{\frac{1}{2}x^2}$

Check $y'(x) = Ce^{\frac{1}{2}x^2} \cdot \frac{1}{2}(2x) = xCe^{+\frac{1}{2}x^2}$

$= xy.$ ✓

2. (5 points). You are given the ODE: $y'(x) = y(x)^2 - 4 = (y(x)-2)(y(x)+2)$.

   a. What is the direction field?

   b. Sketch the direction field mentioning any important properties.

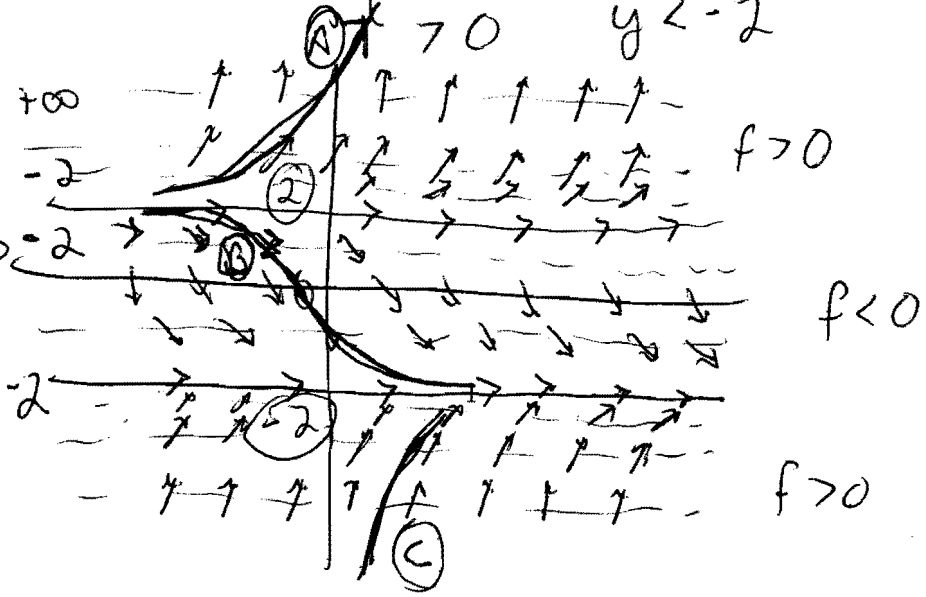
   c. What is the limit of $y(x)$ as $x \to +\infty$ if the solution curve passes through $x = 0$ and $y = 0$?

a) $f(x,y) = y^2 - 4$ • independent of x

• $f(x,y) = 0$ when $y = \pm 2 \Rightarrow$ 2 equilibrium solns $y = \pm 2$.

b) $f(x,y) = (y-2)(y+2)$ $\begin{cases} >0 & y > 2 \\ <0 & -2 < y < 2 \\ >0 & y < -2 \end{cases}$

A going to $+\infty$
B going to -2
C going to -2



c) $\lim_{x \to +\infty} y(x) = -2$