MA214–02 Spring 2011
Quiz #2 - 10 Points
4 February 2011

NAME: Solutions

1. (5 points). Find the most general solution to the ODE:

$$y'(x) + 3xy(x) = x.$$

Integrating factor:

$y' + py = q$, $\frac{d}{dx}(\mu y) = \mu q$; $p(x) = 3x$, $\int p(x)dx = \frac{3}{2}x^2$, $\mu(x) = e^{\frac{3}{2}x^2}$

$\int \mu q\, dx = \int x e^{\frac{3}{2}x^2} dx = \frac{1}{3} e^{\frac{3}{2}x^2} + C$

$u = \frac{3}{2}x^2$

$du = 3x\,dx$ so $\boxed{y(x) = \frac{1}{3} + Ce^{-\frac{3}{2}x^2}}$

You can also separate variables.

$y' = x(1-3y)$

$\int \frac{dy}{1-3y} = \int x\,dx$

2. (5 points). The ODE satisfied by a population $P(t)$ of seals for any time $t \geq 0$ is

$$P'(t) = rP(t) - k,$$

where $r > 0$ is the growth rate and $k > 0$ represents the yearly loss. If the initial population is $P_0 > 0$ find the unique solution. If $P_0 < k/r$, what happens to the population?

Separate: $\frac{dP}{rP-k} = dt$ $\quad \int \frac{dP}{rP-k} = \frac{1}{r}\ln|rP-k| + C = t$

$u = rP-k$

$du = r\,dP$

$\ln|rP-k| = rt + C$

$rP(t) = Ce^{rt} + k$

$\boxed{P(t) = Ce^{rt} + k/r \text{ general soln}}$

Initial Condition: $P(0) = P_0 = C + k/r \Rightarrow C = P_0 - k/r$

$\boxed{P(t) = (P_0 - k/r)e^{rt} + k/r}$

- If $P_0 > k/r$ then $P(t) \uparrow +\infty$
- If $P_0 < k/r$ then $P(t)$ will go to zero and the population dies out.

Check #1: $y' = -\frac{3}{2} \cdot 2x\, Ce^{-\frac{3}{2}x^2}$

$= -3x\,Ce^{-\frac{3}{2}x^2} = -3x\left(y - \frac{1}{3}\right) = -3xy + x$ ✓