MA214–02 Spring 2011
Quiz #4 - 10 Points
4 March 2011

NAME: Solutions

1. (5 points). Consider the ODE:

$$y'' - 5y' + 6y = 0.$$

a. Find the most general solution to the second-order ODE.

b. Find the unique solution satisfying the initial conditions:

$$y(0) = 1, \quad y'(0) = 0.$$

a) $r^2 - 5r + 6 = (r-2)(r-3) = 0 \Rightarrow r_1 = 2,\ r_2 = 3$

$\boxed{y(t) = C_1 e^{2t} + C_2 e^{3t}}$ general soln.

b.) $y'(t) = 2C_1 e^{2t} + 3C_2 e^{3t}$

$\left.\begin{aligned} y(0) &= C_1 + C_2 = 1 \\ y'(0) &= 2C_1 + 3C_2 = 0 \end{aligned}\right\}$ $\begin{aligned} -2C_1 - 2C_2 &= -2 \\ 2C_1 + 3C_2 &= 0 \end{aligned}$ $\begin{aligned} C_2 &= -2 \\ C_1 &= 3 \end{aligned}$

$\boxed{y(t) = 3e^{2t} - 2e^{3t}}$ $y(0) = 3 - 2 = 1$ ✓

$y'(0) = 6 - 6 = 0$ ✓

2. (5 points) Find two independent solutions to the ODE:

$$y'' - 6y' + 9y = 0.$$

Use the Wronskian to check that the solutions are independent.

a) $r^2 - 6r + 9 = (r-3)^2 = 0,\ r = 3$ double root

2 independent solns: $\boxed{y_1(t) = e^{3t} \quad y_2(t) = te^{3t}}$

b.) $y_1'(t) = 3e^{3t} \quad y_2'(t) = (1+3t)e^{3t}$

$$\begin{aligned} W(y_1, y_2)(t) &= y_1(t)y_2'(t) - y_1'(t)y_2(t) \\ &= e^{3t}(1+3t)e^{3t} - 3e^{3t}(te^{3t}) \end{aligned}$$

$$= e^{6t} + 3te^{6t} - 3te^{6t}$$

$\boxed{W(y_1, y_2)(t) = e^{6t} \neq 0 \text{ never} \Rightarrow y_1 \,\&\, y_2 \text{ independent}}$