**MA214–02 Spring 2011**
**Quiz #5 - 10 Points**
**25 March 2011**

NAME: Solutions

1. (4 points) Find two independent **real** solutions to the ODE:

$$y''(t) - 2y'(t) + 2y(t) = 0.$$

Roots: $r^2 - 2r + 2 = 0$

$\frac{1}{2}\{-(-2) \pm [(-2)^2 - 4\cdot 1\cdot 2]^{\frac{1}{2}}\} = 1 \pm \frac{1}{2}[-4]^{\frac{1}{2}} = 1 \pm i$

Solutions: Complex $e^{(1\pm i)t} = e^t(\cos t \pm i \sin t)$

Take linear combinations:

$$\left.\begin{aligned} y_1(t) &= e^t \cos t \\ y_2(t) &= e^t \sin t \end{aligned}\right\}$$ 2 linearly independent real solutions

2. (6 points). Find the most general solution to the nonhomogeneous ODE:

$$y'' - y' - 2y = 4t.$$ $y_g = y_h + y_p$

① $y_h$: $y'' - y' - 2y = 0$

$r^2 - r - 2 = 0 = (r-2)(r+1) \Rightarrow$ 2 roots $r_1 = 2$, $r_2 = -1$

$\boxed{y_1(t) = e^{2t},\ y_2(t) = e^{-t}}$ $\boxed{y_h(t) = C_1 e^{2t} + C_2 e^{-t}}$

② $y_p$: easiest is to guess $y_p(t) = At + B$

$y_p'(t) = A$, $y_p''(t) = 0$

substitute: $0 - A - 2(At + B) = 4t \Rightarrow -2B - A = 0$ & $-2A = 4$

$A = -2$ $B = +1$

$\boxed{y_p(t) = -2t + 1}$ Check: $y_p' = -2$ so $-(-2) - 2(-2t + 1)$

$= 2 + 4t - 2 = 4t$ ✓

If you use Variation of Parameters:

$W(y_1, y_2)(t) = e^{2t}(-e^{-t}) - (2e^{2t})(e^{-t}) = -3e^t$

$u_1(t) = -\int \frac{y_2(t) g(t)}{W(t)} dt = -\int \frac{e^{-t}(4t)}{-3e^t} = \frac{4}{3}\int t e^{-2t} dt = \frac{4}{3}\left(-\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t}\right)$

Integrate by parts once

$u_2(t) = \int \frac{y_1(t) g(t)}{W(t)} dt = \int \frac{e^{2t}(4t)}{-3e^t} = -\frac{4}{3}\int e^t t\, dt = -\frac{4}{3}(te^t - e^t)$ ✓

$y_p(t) = \left(-\frac{2}{3}te^{-2t} - \frac{1}{3}e^{-2t}\right)e^{2t} + \left(-\frac{4}{3}te^t + \frac{4}{3}e^t\right)e^{-t} = -\frac{2}{3}t - \frac{1}{3} - \frac{4}{3}t + \frac{4}{3} = \underline{\underline{-2t + 1}}$