

**MA214 Section 002 Spring 2011**  
**Practice Test #1**  
**14 February 2011**

**REVIEW SESSION: Monday, 14 February, 4PM–5PM, CB 313; Office hours Wednesday, 16 February, 753 POT, 4-5PM. Test #1 on Friday, 18 Feb. in class.**

MATERIAL: Chapter 1; Chapter 2, Sections 1, 2, 3, 5.

1. The velocity of a falling body in air satisfies

$$m \frac{dv}{dt} = -mg - \gamma v,$$

where  $m > 0$  is the mass of the body, the constant  $g > 0$  is the gravitational constant, and  $\gamma > 0$  is the drag due to air resistance.

- (a) Find the most general solution to the equation.
  - (b) Suppose that  $g = 10m/sec/sec$  (an approximation), the body has mass  $m = 10kg$ , and the drag coefficient is  $\gamma = 2kg/sec$ . If  $v(0) = 0$ , find the body's velocity at any time  $t > 0$ .
  - (c) The body is dropped from a height of  $300m$ . Write the equation for the position  $x(t)$ , measured positively from  $x = 0$ .
  - (d) How long does it take for the body to hit the ground? Simply write the equation satisfied by the time  $T$ .
2. Solve the ODE:

$$y'' + 6ty' = 6t, \quad y'(t = 0) = 1, \quad y(t = 0) = 1,$$

by reduction of order. Let  $u(t) = y'(t)$  and find the solution  $u(t)$  satisfying the initial condition. Then find the solution  $y(t)$ . Check your answer.

3. Suppose a sum  $S_0$  is invested at an annual rate of return  $r$  compounded continuously. Find the time  $T$  it takes for the original sum to double as a function of  $r$ . What must  $r$  be if the sum doubles in 8 years?
4. Clearly identify the direction field of the ODE:  $y' = 4 - 2y$ . Sketch the direction field and describe its main features. What is  $\lim_{t \rightarrow +\infty} y(t)$ ?
5. A tank initially contains 100 liters of pure water. A stream of polluted water with a concentration of  $\gamma = 5$  grams per liter of mercury is poured into the tank at a rate of one liter per minute. It flows out at the same rate. How much mercury is in the tank after  $100 \ln 2$  minutes?
6. Find the general solution to the ODE:

$$ty'(t) + 3y(t) = 6t^2.$$

Find the unique solution to the ODE with initial condition  $y(1) = 0$ .

7. Solve the ODE:

$$y' = \frac{3x^2 - 1}{3 + 2y}.$$

8. The population of bacteria  $P(t)$  satisfies the ODE:

$$\frac{dP}{dt} = r \left( 1 - \frac{P}{K} \right) P,$$

for  $t \geq 0$ . The constants  $r > 0$  and  $K > 0$  represent the growth rate and the saturation level, respectively.

- a. Find the population if the initial population  $P(0)$  satisfies  $0 < P(0) < K$ .
  - b. What is  $\lim_{t \rightarrow \infty} P(t)$  in case (a.)?
9. A room contains 100 cubic meters ( $m^3$ ) of gas with an initial concentration of  $5g/m^3$  of a poisonous gas. Air enters the room at a rate of  $20m^3/min$  and has a concentration of  $5e^{-t/4} g/m^3$  of poison. Air flows out of the room at the same rate. Find the quantity  $Q(t)$  of poison in the room at any  $t > 0$ . Recall that

$$\frac{dQ}{dt} = (\text{rate in}) - (\text{rate out}),$$

and the rate in or out is the flow rate times the concentration.