MA214 Spring 2010
Practice Test #2: Test #2 on Friday, 9 April 2010

REVIEW SESSION: Wednesday, 7 April, 4PM–5PM, 753 POT

MATERIAL: All of Chapter 3 that we covered in class. No Laplace transforms on this test.

1. Show that $y_1(x) = x$ and $y_2(x) = xe^x$ form an independent set of solutions to the ODE:
$$x^2 y'' - x(x+2)y' + (x+2)y = 0, \quad x > 0.$$
2. Are the two functions $w_1(t) = t(t+1)$ and $w_2(t) = t^2$ independent on the real line?
3. Find the most general solution to the ODE:
$$y''(t) - ty'(t) = t,$$
using the substitution $u(t) = y'(t)$.
4. Find the unique solution to the initial value problem
$$y'' + 5y' - 6y = 0,$$
with the conditions $y(0) = 1$ and $y'(0) = 0$.
5. Use Abel's formula to compute the Wronskian of two solutions to the ODE
$$2t^2 y'' + 3ty' - y = 0, \quad t > 0.$$
6. Use variation of parameters to find a particular solution to
$$y'' + 4y' + 4y = t^{-2} e^{-2t}.$$
7. Find the most general real solution to the ODE:
$$y'' + 2y' + 2y = 4t.$$
8. Find the unique solution to the initial value problem
$$4y'' + 12y' + 9y = 0,$$
with $y(0) = 1$, and $y'(0) = 1$.
9. Find the unique solution to the initial value problem
$$y'' + 4y = 3\sin 2t,$$
with $y(0) = 2$ and $y'(0) = -1$.
10. Consider a driven, undamped harmonic oscillator described by the ODE
$$u'' + 2u = 2\cos \omega t.$$
What is the natural frequency? Find the solutions for ω not equal to, and for ω equal to, the natural frequency, when the initial conditions are $u(0) = 0$ and $u'(0) = 0$.

11. What is a set of independent *real* solutions for the damped, undriven oscillator described by
$$u'' + 4u' + 4u = 0.$$
What is the unique solution to this ODE with initial conditions $u(0) = 1$ and $u'(0) = 0$? How long does it take for the amplitude to decrease to one-half of its initial value?

12. Use the method of undetermined coefficients to find the unique solution of
$$y'' + 4y = t, \quad y(0) = 1, y'(0) = 0.$$