MA214 002 Spring 2011 Practice Test #2: Test #2 on Friday, 8 April 2011

We will review on Wednesday, 6 April.

MATERIAL: All of Chapter 3 that we covered in class and Euler's method.

1. Show that $y_1(x) = x$ and $y_2(x) = xe^x$ form an independent set of solutions to the ODE:

$$x^{2}y'' - x(x+2)y' + (x+2)y = 0, \quad x > 0$$

- 2. Are the two functions $w_1(t) = t(t+1)$ and $w_2(t) = t^2$ independent on the real line?
- 3. Find the most general real solution to the ODE:

$$y''(t) - 2y'(t) + 2y = 0.$$

4. Find the unique solution to the initial value problem

$$y'' + 5y' - 6y = 0,$$

with the conditions y(0) = 1 and y'(0) = 0.

5. Use Abel's formula to compute the Wronskian of two solutions to the ODE

$$2t^2y'' + 3ty' - y = 0, \quad t > 0.$$

6. Use variation of parameters to find a particular solution to

$$y'' + 4y' + 4y = t^{-2}e^{-2t}.$$

7. Find the most general real solution to the ODE:

$$y'' + 2y' + 2y = 4t.$$

8. Find the unique solution to the initial value problem

$$4y'' + 12y' + 9y = 0,$$

with y(0) = 1, and y'(0) = 1.

9. Find the unique solution to the initial value problem

$$y'' + 4y = 3\sin 2t,$$

with y(0) = 2 and y'(0) = -1.

10. Describe Euler's method for the ODE: y'(t) = f(t, y). Let f(t, y) = y. Find the exact solution with the initial condition y(0) = 1. Now take a step size h = 1 and compute the approximation at t = 3. What is the exact value? 11. Consider a driven, undamped harmonic oscillator described by the ODE

$$u'' + 2u = 2\cos\omega t.$$

What is the natural frequency? Find the solutions for ω not equal to, and for ω equal to, the natural frequency, when the initial conditions are u(0) = 0 and u'(0) = 0.

12. What is a set of independent *real* solutions for the damped, undriven oscillator described by

$$u'' + 4u' + 4u = 0.$$

What is the unique solution to this ODE with initial conditions u(0) = 1and u'(0) = 0? How long does it take for the amplitude to decrease to one-half of its initial value?

13. Use the method of undetermined coefficients (intelligent guessing) to find the unique solution of

$$y'' + 4y = t$$
, $y(0) = 1, y'(0) = 0$.