MA214–003 Fall 2008
Some Useful Results: Complex Numbers, Exponentials, and Euler's Formula
September 26, 2008

Exponential and Log Functions

1. $e^a e^b = e^{a+b}$
2. $e^{ab} = (e^a)^b = (e^b)^a$
3. $c\, logb = log(b^c), \quad b > 0$
4. $log(a/b) = loga - logb, \quad a, b > 0$
5. $log(ab) = loga + logb, \quad a, b > 0$
6. $a^b = e^{bloga}, \quad a > 0$

Complex Numbers

1. **Definition.** $z = x + iy, \;\; x, y$ real numbers. We call $x = \mathrm{Re}z$, the real part of z, and $y = \mathrm{Im}z$, the imaginary part of z. Note that both x and y are real. The imaginary number i is $i = \sqrt{-1}$, so that $i^2 = -1$. Sometimes engineers write j for i.
2. **Addition.** Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, then $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$.
3. **Multiplication.** As above, $z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$.
4. **Complex Conjugation.** If $z = x + iy$, then its complex conjugate $\overline{z}$, is $\overline{z} = x - iy$. Sometimes the complex conjugate is denoted by z^*.
5. **Modulus.** For $z = x + iy$, we have $|z|^2 = \overline{z}z = x^2 + y^2$. This is a real number.
6. **Euler's Formula.** For a real number θ, we have
$$e^{i\theta} = cos\theta + isin\theta.$$
This can be proved using the expansion
$$e^u = \sum_{j=0}^{\infty} \frac{u^j}{j!},$$
and the Taylor expansions for the sine and cosine functions.
7. **A Related Formula.** For $z = x + iy$, we have
$$e^z = e^{x+iy} = e^x e^{iy} = e^x(cosy + isiny).$$