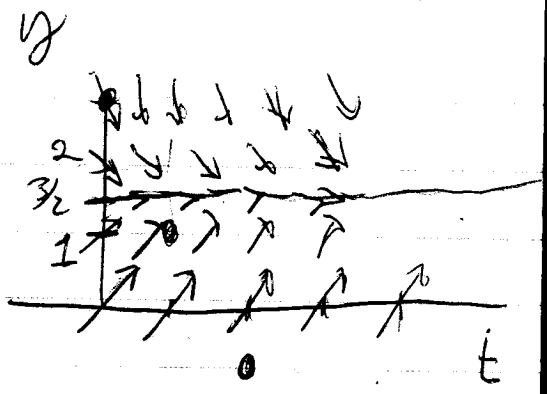
MA 214 PS #1 (1)

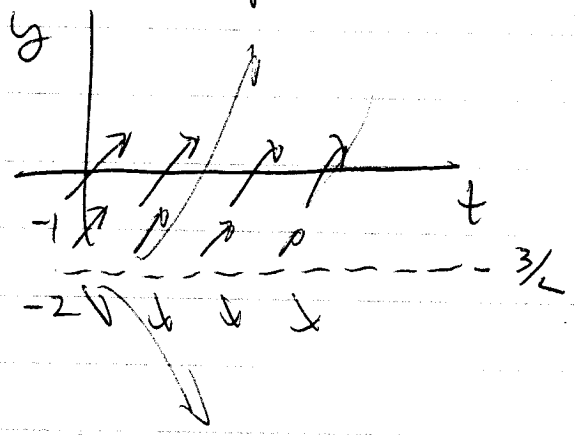
pg 7 Direction field



1. $y' = 3 - 2y$ $\quad F(t,y) = 3 - 2y$ independent of t

Soln $3/2$ $\quad (t,y)$

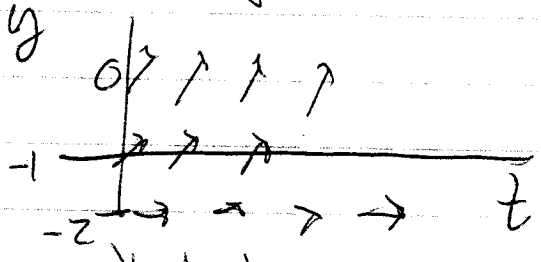
$y(t) \to 3/2$ evaluate $f(t,y)$ & take unit vector ⊥ this slope.



3. $y' = 3 + 2y$

$f(t,y) = 3 + 2y$ indep of t

$y(t) \to +\infty \quad -3/2$



6. $y' = y + 2$

$f(t,y) = y + 2$

$y(t) \to +\infty$ unless $y_0 = -2$ in which case $y(t) = -2$ for all t

pg 15-16 2. $y' = y - 5$ $\quad \frac{dy}{y-5} = dt$

$\ln|y-5| = t + C$

$y - 5 = Ce^t$

$y(t) = 5 + Ce^t$

$y(0) = 5 + C = y_0$

$C = y_0 - 5$

$y(t) = 5(1 - e^t) + y_0 e^t$

depends on $y_0 \gtrless 5$

4. $y'(t) = y' - 0 = ay - b = a(y + y_0) - b$

5. $\frac{dy}{dt} = ay$ $\quad y_1(t) = Ce^{at}$

Suppose $y(t) = Ce^{at} + k$

$y'(t) = aCe^{at} = a(y-k)$

$= ay + ak$

$b = ak$

$\boxed{k = b/a}$

8. $p(t) = p_0 e^{rt}$ soln to $p' = rp$

a) Pop $p(30) = 2p_0 = p_0 e^{r \cdot 30}$ doubles in 30 days

so $2 = e^{30r}$ $\ln 2 = 30r$ $r = \ln 2/30$ $\quad p_0 \to 2p_0$ in 30 days

b) Pop $p(N) = 2p_0 = p_0 e^{rN}$

$\log 2 = rN$

$r = \log 2/N$.

9. $V' = 9.8 - \frac{V}{5}$ $\quad V(t) = 49 + ce^{-t/5}$

$v(0) = 0 \Rightarrow c = -49$

$V(t) = 49(1 - e^{-t/5})$

limiting velocity $\lim_{t \to +\infty} V(t) = \underbrace{49}_{V_\infty}$ m/sec

a) $V(t_0) = 0.98 V_\infty = 49(1 - e^{-t_0/5})$

$0.02 V_\infty = 1 - e^{-t_0/5}$

$0.98 = 1 - e^{-t_0/5}$

$e^{-t_0/5} = 0.02$ $\quad -t_0/5 = \log 0.02$

$t_0 = -5 \log 0.02$

b) $x(t) = \int V(t)\,dt = 49t + 245e^{-t/5} + \cancel{k_0}$

Compute $x(t_0) - x(0)$

PSI

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1 order 2 linear
2 order 2 NL
3 order 4 L
4 order 1 NL
5. order 2 NL

9. $ty' - y = t^2$ 1st order linear nonhomog.

$y = 3t + t^2$

$y' = 3 + 2t$

$ty' = 3t + 2t^2 \stackrel{?}{=} t^2 + y = 2t^2 + 3t$ soln. ✓

10. $y'''' + 4y'' + 3y = t$

$y_1 = t/3$, $y_1' = 1/3$, $y_1'' = 0$ } $3 \cdot t/3 = t$ ok

$y_2(t) = e^{-t} + t/3$
$y_2'(t) = -e^{-t} + 1/3$
$y_2''(t) = e^{-t}$
$y_2'''(t) = -e^{-t}$
$y_2''''(t) = e^{-t}$

$e^{-t} + (4 \cdot e^{-t}) + 3e^{-t} + t$
$= t$ ✓

15. $y' + 2y = 0$ $y = e^{rt}$ $y' = re^{rt}$

so $y' + 2y = (r+2)e^{rt} = 0$

$\Rightarrow r = -2$

16. $y'' - y = 0$ $y = e^{rt}$ $y' = re^{rt}$ $y'' = r^2 e^{rt}$

$(r^2 - 1)e^{rt} = 0$

$r = \pm 1$ 2 soln e^t and e^{-t}.