

Solutions for PS 2

pg 39 3.

$$y' + y = te^{-t} + 1$$

$$p(t) = 1$$

$$g(t) = te^{-t} + 1$$

Integrating factor

$$\mu' = p\mu$$

$$\mu' = \mu \text{ so } \mu = e^t$$

$$\frac{d}{dt}(y\mu) = \frac{d}{dt}(e^t y) = e^t(te^{-t} + 1) = t + e^t$$

$$\text{Integrate: } e^t y(t) = \int (t + e^t) dt = \frac{1}{2}t^2 + e^t + C$$

$$y(t) = \frac{1}{2}t^2 e^{-t} + 1 + e^{-t}C$$

$$\text{Check: } y' = te^{-t} - \frac{1}{2}t^2 e^{-t} - e^{-t}C$$

$$y' + y = te^{-t} + 1 \quad \checkmark$$

5.

$$y' - 2y = 3e^t$$

$$p = -2$$

$$g = 3e^t$$

$$\mu' = -2\mu \Rightarrow \mu = e^{-2t}$$

$$\frac{d}{dt}(e^{-2t} y) = e^{-2t} \cdot 3e^t = 3e^{-t}$$

$$\Rightarrow e^{-2t} y = -3e^{-t} + C$$

$$y(t) = -3e^t + Ce^{2t}$$

$$\text{Check: } y' = -3e^t + 2Ce^{2t}$$

$$y' - 2y = 3e^t \quad \checkmark$$

14.

$$y' + 2y = te^{-2t}$$

$$p = 2$$

$$g = te^{-2t}$$

$$\mu = e^{2t}$$

$$y(1) = 0$$

$$(\text{check: } \mu' = 2\mu)$$

$$\frac{d}{dt}(e^{2t} y) = t \Rightarrow e^{2t} y = \frac{1}{2}t^2 + C$$

P52

(p2)

$$y(t) = \frac{1}{2}t^2 e^{-2t} + C e^{-2t}$$

check $y' = t e^{-2t} - t^2 e^{-2t} - 2C e^{-2t}$

$$y' + 2y = t e^{-2t} \quad \checkmark$$

Initial Cond.

$$y(0) = 0 = \frac{1}{2}e^{-2} + C e^{-2}$$

$$C e^{-2} = -\frac{1}{2}e^{-2} \quad C = -\frac{1}{2}$$

$$y(t) = \frac{1}{2}t^2 e^{-2t} - \frac{1}{2}e^{-2t}$$

pg 47.2. $y' = \frac{x^2}{y(1+x^3)}$ or $y dy = \left(\frac{x^2}{1+x^3} \right) dx$

let $u = 1+x^3$
 $du = 3x^2 dx$

$$\frac{1}{2}y^2 + C = \frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \ln |x^3 + 1|$$

$$\frac{1}{2}y^2 = \frac{1}{3} \ln |x^3 + 1| + C$$

10. $y' = \frac{1-2x}{y}$ & $y(1) = -2$. $\int y dy = \int (1-2x) dx$

$$\frac{1}{2}y^2 + C = x - x^2$$

$y(1) = -2$ so

$$2 + C = 0 \quad C = -2$$

$$\frac{1}{2}y^2 = 2 + x - x^2$$

12. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $r(1) = 2$: $\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$ or $-r^{-1} + C = \ln |\theta|$

$$-\frac{1}{r(1)} + C = \ln 1 = 0 \quad \text{or} \quad C = \frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{r} = \ln |\theta|$$

23. $y' = 2y^2 + x^2 = y^2(2+x)$ and $y(0) = 1$

$$\frac{dy}{y^2} = (2+x) dx$$

$$-y^{-1} = 2x + \frac{1}{2}x^2 + C$$

$$-1 = C \quad \text{Critical pt: } y' = 0$$

$$x = -2 \quad \text{or} \quad y = 0$$

$$\frac{-1}{2x + \frac{1}{2}x^2 + 1} = y(x)$$