MA214 section 3.4.

Sec. 3.4 Complex Roots

pg164 2. $e^{2-3i} = e^2 e^{-3i} = e^2(\cos 3 - i\sin 3) = \underbrace{e^2\cos 3}_{\text{Real part}} + i\underbrace{(-e^2\sin 3)}_{\text{Im part}}$

4. $e^{2-i\pi/2} = e^2(\cos\pi/2 - i\sin\pi/2) = -e^2\sin\frac{\pi}{2} i = -e^2 i$

$Re(e^{2-i\pi/2}) = 0 \quad Im(e^{2-i\pi/2}) = -e^2$.

6. $\pi^{-1+2i} = e^{(-1+2i)\log\pi} = e^{-\ln\pi} e^{2i\ln\pi}$

$= \pi^{-1}[\cos(\ln\pi^2) + i\sin(\ln\pi^2)]$

$e^{2i\ln\pi} = e^{i\ln\pi^2} = \cos(\ln\pi^2) + i\sin(\ln\pi^2)$

8. $y'' - 2y' + 6y = 0 \qquad r^2 - 2r + 6 = 0$

$b^2 - 4ac = 4 - 24 = -20 = -4\cdot5$

roots: $\frac{2 \pm [-4.5]^{\frac12}}{2} = 1 \pm i\sqrt5$

$e^{(1+i\sqrt5)t},\ e^{(1-i\sqrt5)t}$

Take $Re\ e^t e^{i\sqrt5 t} = e^t\cos\sqrt5 t$

$Im\ e^t e^{i\sqrt5 t} = e^t\sin\sqrt5 t$ } 2 independent solution.

$y(t) = C_1 e^t\cos\sqrt5 t + C_2 e^t \sin\sqrt5 t$.

10. $y'' + 2y' + 2y = 0 \qquad r^2 + 2r + 2 = 0 \qquad r_1 = -1-it$

$y_1(t) = e^{-t}(\cos t - i\sin t) \qquad r_2 = -1+it$

$y_2(t) = e^{-t}(\cos t + i\sin t)$

$Re\,y_1 = e^{-t}\cos t \quad Im\,y_1 = e^{-t}\sin t$ both solutions & Independent

$y(t) = C_1 e^{-t}\cos t + C_2 e^{-t}\sin t$

12. $4y'' + 9y = 0 \quad y'' + (3/2)^2 y = 0 \Rightarrow y_1(t) = \cos(3/2 t) \qquad y_2(t) = \sin(3/2 t)$

$y(t) = C_1\cos(3/2 t) + C_2\sin(3/2 t)$

Harmonic Osc! $\quad r^2 + (3/2)^2 = 0 \quad r = \pm i\, 3/2$

$e^{\pm rt} = e^{\pm i 3/2 t} = \cos(3/2 t) \pm i\sin(3/2 t)$

18. $y'' + 4y' + 5y = 0 \quad r^2 + 4r + 5 = 0 \qquad \frac{-4 \pm [16-20]^{\frac12}}{2} = \underline{\underline{-2 \pm i}}$

$y(t) = C_1 e^{-2t}\cos t + C_2 e^{-2t}\sin t \qquad y(0) = 1\ \&\ y'(0) = 0.$

$y(0) = 1 = C_1 \qquad y'(0) = -2 + C_2 = 0 \quad C_2 = 2$