1. Find the unique solution of the initial value problem:

\[ 2xy^2 y'(x) = 1, \quad y(1) = 1. \]

State on what interval the solution exists.

2. Simplify: \( e^{2+i\pi}, \quad 3^2, (5i + 3)(-4i). \)

3. Find the unique solution of the initial value problem:

\[ y''(t) - 5y'(t) + 6y(t) = 0, \]

with \( y(0) = 0 \) and \( y'(0) = 1. \)

4. The velocity of a falling body in air satisfies

\[ m \frac{dv}{dt} = mg - \gamma v, \]

where \( m > 0 \) is the mass of the body, the constant \( g > 0 \) is the gravitational constant, and \( \gamma > 0 \) is the drag due to air resistance.

(a) Find the most general solution to the equation.
(b) Suppose that \( g = 10 \text{m/sec/sec} \) (an approximation), the body has mass \( m = 10 \text{kg} \), and the drag coefficient is \( \gamma = 2 \text{kg/sec} \). If \( v(0) = 0 \), find the body’s velocity at any time \( t > 0 \).
(c) The body is dropped from a height of 300m. Write the equation for the position \( x(t) \), measured positively from \( x = 0 \).
(d) How long does it take for the body to hit the ground? Simply write the equation satisfied by the time \( T \).

5. Find the general solution to the ODE:

\[ ty''(t) + 3y(t) = 6t^2. \]

Find the unique solution to the ODE with initial condition \( y(1) = 0. \)

6. Solve the ODE:

\[ y' = \frac{3x^2 - 1}{3 + 2y}. \]

Does the ODE have a unique solution if \( y(0) = -3/2 \)? State why or why not. Does the ODE have a unique solution with \( y(0) = 1 \)? If yes, find it. For what interval about \( x = 0 \) is it valid?

7. Suppose a sum \( S_0 \) is invested at an annual rate of return \( r \) compounded continuously. Find the time \( T \) it takes for the original sum to double as a function of \( r \). What must \( r \) be if the sum doubles in 8 years?
8. Find the unique solution to the ODE using only real functions:

\[ y''(t) + 16y(t) = 0, \quad y(0) = 1, \quad y'(0) = 0. \]

9. Find the most general solution to the ODE

\[
\left(\frac{y}{x} + 6x\right) + (\ln x - 2)\frac{dy}{dx} = 0, \quad x > 0.
\]