MA214 Section 003 Fall 2008
Practice Test #1 - 100 Points
8 October 2008

REVIEW SESSION: Wednesday, 8 October, 5PM–6PM, CB 205

MATERIAL: Chapter 1; Chapter 2, Sections 1, 2, 3, 4, 6; Chapter 3, Section 1, 2, 3, 4.

1. Find the unique solution of the initial value problem:
$$2yx^2y'(x) = 1, \quad y(1) = 1.$$
State on what interval the solution exists.

2. Simplify: $e^{2+i\pi}$, 3^{2i}, $(5i+3)(-4i)$.

3. Find the unique solution of the initial value problem:
$$y''(t) - 5y'(t) + 6y(t) = 0,$$
with $y(0) = 0$ and $y'(0) = 1$.

4. The velocity of a falling body in air satisfies
$$m\frac{dv}{dt} = mg - \gamma v,$$
where $m > 0$ is the mass of the body, the constant $g > 0$ is the gravitational constant, and $\gamma > 0$ is the drag due to air resistance.
   (a) Find the most general solution to the equation.
   (b) Suppose that $g = 10m/sec/sec$ (an approximation), the body has mass $m = 10kg$, and the drag coefficient is $\gamma = 2kg/sec$. If $v(0) = 0$, find the body's velocity at any time $t > 0$.
   (c) The body is dropped from a height of $300m$. Write the equation for the position $x(t)$, measured positively from $x = 0$.
   (d) How long does it take for the body to hit the ground? Simply write the equation satisfied by the time T.

5. Find the general solution to the ODE:
$$ty'(t) + 3y(t) = 6t^2.$$
Find the unique solution to the ODE with initial condition $y(1) = 0$.

6. Solve the ODE:
$$y' = \frac{3x^2 - 1}{3 + 2y}.$$
Does the ODE have a unique solution if $y(0) = -3/2$? State why or why not. Does the ODE have a unique solution with $y(0) = 1$? If yes, find it. For what interval about $x = 0$ is it valid?

7. Suppose a sum S_0 is invested at an annual rate of return r compounded continuously. Find the time T it takes for the original sum to double as a function of r. What must r be if the sum doubles in 8 years?

8. Find the unique solution to the ODE using only real functions:

$$y''(t) + 16y(t) = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

9. Find the most general solution to the ODE

$$(y/x + 6x) + (\ln x - 2)\frac{dy}{dx} = 0, \quad x > 0.$$