

MA214 Fall 2008
Practice Test #2

MATERIAL: Chapter 3, sections 3.5, 3.6, 3.7, 3.8, 3.9; Chapter 6, sections 6.1–6.4. We'll do the word problems of sections 3.8–3.9 and section 6.5 after the Thanksgiving break so this material will NOT be on test 2.

1. Show that $y_1(x) = x$ and $y_2(x) = xe^x$ form an independent set of solutions to the ODE:

$$x^2y'' - x(x+2)y' + (x+2)y = 0, \quad x > 0.$$

2. Are the two functions $w_1(t) = t(t+1)$ and $w_2(t) = t^2$ independent on the real line?
3. Find the most general solution to the ODE:

$$y''(t) - ty'(t) = t,$$

using the substitution $u(t) = y'(t)$.

4. Find the unique solution to the initial value problem

$$y'' + 5y' - 6y = 0,$$

with the conditions $y(0) = 1$ and $y'(0) = 0$.

5. Compute the Laplace transform of $f(t) = e^{2t}t^2$.
6. Use Abel's formula to compute the Wronskian of two solutions to the ODE

$$2t^2y'' + 3ty' - y = 0, \quad t > 0.$$

7. Use variation of parameters to find a particular solution to

$$y'' + 4y' + 4y = t^{-2}e^{-2t}.$$

8. Find the Laplace transform of the following functions:

$$f(t) = t \sin 4t \text{ (use integration by parts)}$$

$$g(t) = \begin{cases} 0 & 0 \leq t \leq 1 \\ (t-1) & t \leq 1 \leq 2 \\ 1 & t \geq 2 \end{cases}$$

9. Find the most general real solution to the ODE:

$$y'' + 2y' + 2y = 4t.$$

10. Find the unique solution to the initial value problem

$$4y'' + 12y' + 9y = 0,$$

with $y(0) = 1$, and $y'(0) = 1$.

11. Find the unique solution to the initial value problem

$$y'' + 4y = 3 \sin 2t,$$

with $y(0) = 2$ and $y'(0) = -1$.

12. Consider a driven, undamped harmonic oscillator described by the ODE

$$u'' + 2u = 2 \cos \omega t.$$

What is the natural frequency? Find the solutions for ω not equal to, and for ω equal to, the natural frequency, when the initial conditions are $u(0) = 0$ and $u'(0) = 0$.

13. What is a set of independent *real* solutions for the damped, undriven oscillator described by

$$u'' + 4u' + 4u = 0.$$

What is the unique solution to this ODE with initial conditions $u(0) = 1$ and $u'(0) = 0$? How long does it take for the amplitude to decrease to one-half of its initial value?

14. Use the Laplace transform method to solve the initial value problem

$$y'' - 2y' - 2y = u_2(t), \quad y(0) = 2, y'(0) = 0.$$

15. Use the method of undetermined coefficients to find the unique solution of

$$y'' + 4y = t, \quad y(0) = 1, y'(0) = 0.$$

16. Find the inverse Laplace transform of the functions:

$$F(s) = \frac{e^{-\pi s}}{s(s^2 + 1)}, \quad F(s) = \frac{2s - 3}{s^2 + 2s + 1}.$$