Quiz #1 - 10 Points
23 January 2009

NAME: Solutions

1. (5 points). Find the most general solution to

$$y' = 2y + 1.$$

Separate Variables

$$\int \frac{dy}{2y+1} = \int dt = t + C_1$$

$$\int \frac{dy}{2y+1} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|2y+1| + C_0$$

$u = 2y+1$

$du = 2dy$

$\underline{\text{so}}$ $\quad \frac{1}{2}\ln|2y+1| = t + C_0$

$$2y+1 = C_2 e^{2t}$$

$$\boxed{y(t) = C_3 e^{2t} - \frac{1}{2}}$$

Check: $y' = 2C_3 e^{2t} = 2(y + \frac{1}{2}) = 2y + 1$ ✓

2. (5 points). Clearly identify the direction field of the ODE: $y' = 3 - 2y$. Sketch the direction field. What is $\lim_{t\to+\infty} y(t)$?

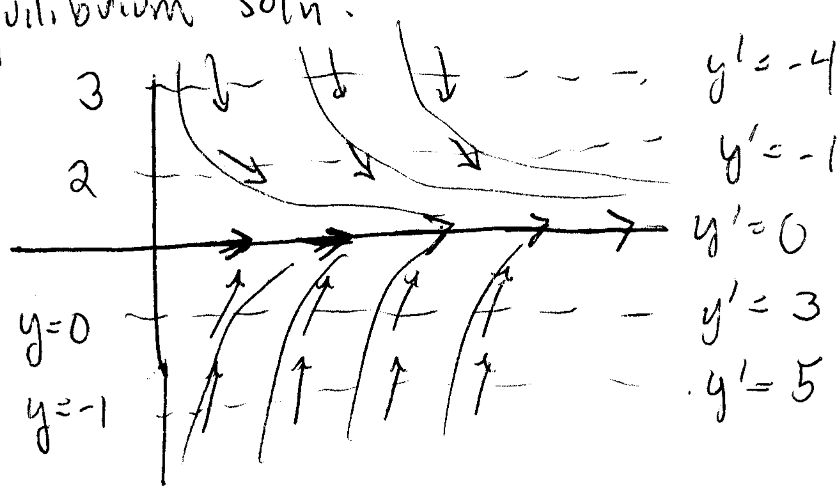
$f(t,y) = 3 - 2y$ independent of t

$y = 3/2$ equilibrium soln.

$y > 3/2 \Rightarrow y' < 0$ $\quad y = \frac{3}{2}$

$y < \frac{3}{2}$

$y' > 0$



$$\boxed{\lim_{t\to\infty} y(t) = \frac{3}{2}}$$