

NAME: Solutions

1. (6 points). Find the most general solution to

$$y' - 3y = 5.$$

Separate Variables:  $\frac{dy}{3y+5} = dt$

Integrate:  $u = 3y+5$   
 $du = 3 dy$

$$\frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \ln |3y+5| = t + C$$

$$3y+5 = Ce^{3t}$$

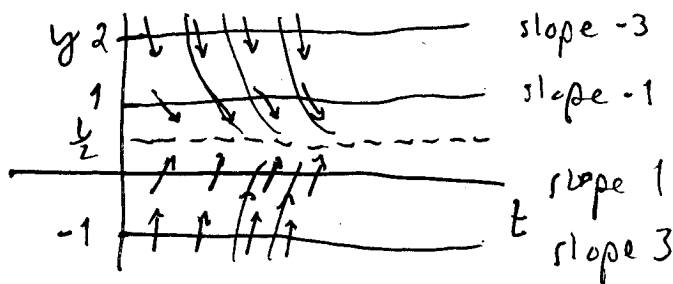
$$y = -\frac{5}{3} + Ce^{3t}$$

check:  $y' = 3Ce^{3t}$   
 $y' - 3y = 3Ce^{3t} - 3(-\frac{5}{3} + Ce^{3t}) + 5$   
 $= 5 \checkmark$

2. (4 points). Clearly identify the direction field of the ODE:  $y' + 2y = 1$ . Sketch the direction field. What is  $\lim_{t \rightarrow +\infty} y(t)$ ?

ODE:  $y' = 1 - 2y$

Direction field:  $f(t, y) = 1 - 2y$  independent of  $t$



solution curves

$$f(t, \frac{1}{2}) = 0$$

$$y \rightarrow \frac{1}{2} \quad f(t, y) < 0$$

$$\lim_{t \rightarrow \infty} y(t) = \frac{1}{2} \quad \text{equilibrium soln.}$$