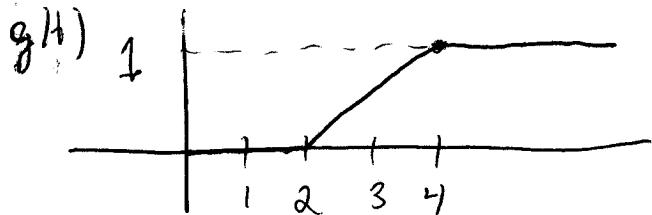


NAME: Solutions

1. Use the Laplace transform method to find the unique solution ODE:

$$u''(t) + 9u(t) = \begin{cases} 0 & 0 \leq t < 2 \\ (t-2)/2 & 2 \leq t < 4 \\ 1 & t \geq 4, \end{cases} \quad g(t)$$

with initial conditions $u(0) = u'(0) = 0$. Formulas are on the board.



$$\begin{aligned} g(t) &= u_2(t)\left(\frac{t-2}{2}\right) + u_4(t) = u_4(t)\left(\frac{t-2}{2}\right) \\ &= \frac{1}{2}u_2(t)(t-2) + \frac{1}{2}u_4(t)(4-t) \end{aligned}$$

$$(s^2 + 9)(\mathcal{L}u)(s) = (\mathcal{L}g)(s) = \frac{1}{2} \frac{e^{-2s}}{s^2} - \frac{1}{2} \frac{e^{-4s}}{s^2}$$

using the LT formula for $u_c(t)f(t-c)$ with $f(t)=t$

$$(\mathcal{L}u)(s) = \frac{1}{2} \left(\frac{e^{-2s}}{s^2(s^2+9)} - \frac{e^{-4s}}{s^2(s^2+9)} \right)$$

Partial fraction: $\frac{1}{s^2(s^2+9)} = \frac{A}{s^2} + \frac{B}{s^2+9}$ so $A = \frac{1}{9}$ $B = -\frac{1}{9}$

$$(\mathcal{L}u)(s) = \frac{1}{18} \left\{ \frac{e^{-2s}}{s^2} - \frac{e^{-2s}}{s^2+9} + \frac{e^{-4s}}{s^2} + \frac{e^{-4s}}{s^2+9} \right\}$$

$$\begin{aligned} u(t) &= \frac{1}{18} \left\{ u_2(t)(t-2) - u_4(t)(t-4) - \frac{1}{3}u_2(t) \sin 3(t-2) \right. \\ &\quad \left. + \frac{1}{3}u_4(t) \sin 3(t-4) \right\}. \end{aligned}$$