MA214–03 Fall 2008
Quiz #10 - 10 Points
5 December 2008

NAME: Solutions

1. (10 points). Find the unique solution to the ODE for $t \geq 0$:

$$y''(t) + 2y'(t) + y(t) = \delta(t-1), \quad y(0) = 0 = y'(0).$$

$$(s^2 + 2s + 1)(\mathcal{L}y)(s) = e^{-s}$$

$$(\mathcal{L}y)(s) = \frac{e^{-s}}{(s+1)^2} = e^{-s} H(s)$$

Recall: $f(t) = t \qquad (\mathcal{L}f)(s) = \frac{1}{s^2}$

so $H(s) = \frac{1}{(s+1)^2} = \frac{1}{(s-(-1))^2}$

$$(\mathcal{L}^{-1}H)(t) = e^{-t}t = te^{-t}$$

Then:

$$\boxed{y(t) = u_1(t)(t-1)e^{-(t-1)}}$$