Quiz #2 - 10 Points
2 February 2009

NAME: Solutions

1. (5 points). Find the unique solution of the initial value problem for $x \geq 0$:

$$y' = \frac{x^2+1}{4y^3}, \quad y(0) = \frac{1}{\sqrt{2}}.$$

Separate variables: $4y^3\,dy = (x^2+1)\,dx$

Integrate: $\boxed{y^4 = \frac{1}{3}x^3 + x + C}$ general soln.

For $y(0) = \frac{1}{\sqrt{2}}$ set $x=0$ $\left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} = C$

so $y(x) = \sqrt[4]{\frac{1}{3}x^3 + x + \frac{1}{4}}$, $x \geq 0$

2. (5 points). Find the most general solution to

$$y' + \frac{2t}{t^2+1}\cancel{y} = 1.$$

$p(t) = \frac{2t}{t^2+1}$

Integrating factor:

$\mu = e^{\int p(t)}$

$\int p(t)\,dt = \int \frac{2t}{t^2+1}\,dt = \ln(t^2+1)$

$u = t^2+1 \quad du = 2t\,dt$

$\mu = e^{\ln(t^2+1)} = t^2+1$

$\frac{d}{dt}\left((t^2+1)y\right) = (t^2+1)$ so $(t^2+1)\,y(t) = \frac{1}{3}t^3 + t + C$

$$\boxed{y(t) = \left(\frac{\frac{1}{3}t^3 + t}{t^2+1}\right) + \frac{C}{t^2+1}}$$