MA214–03 Fall 2008
Quiz #2 - 10 Points
12 September 2008

NAME: Solutions

1. (5 points). Find a positive solution of the initial value problem for $x \geq 0$:

$$y' = \frac{x^2+1}{4y^3}, \quad y(0) = \frac{1}{\sqrt{2}}.$$

Separate and integrate: $\int 4y^3\,dy = \int (x^2+1)\,dx$

$y^4 = \frac{1}{3}x^3 + x + C$

Initial Condition: $y^4(0) = \frac{1}{4} = C$ so $y^4 = \frac{1}{3}x^3 + x + \frac{1}{4}$. For $x \geq 0$, this is positive so

$$\boxed{y(x) = \left[\frac{1}{3}x^3 + x + \frac{1}{4}\right]^{\frac{1}{4}}, \; x \geq 0.}$$

Check: $y' = \frac{1}{4}y^{-3}(x^2+1) = \frac{x^2+1}{4y^3}$ ✓

2. (5 points). Find the most general solution to the ODE:

$$y'(x) - 2xy(x) = x.$$

Integrating factor: $p(x) = -2x \qquad \mu' = p(x)\mu$

$\Rightarrow \mu = e^{-x^2}$

$\frac{d}{dx}(\mu y) = xe^{-x^2}$ or $\mu y = \int xe^{-x^2}dx = -\frac{1}{2}e^{-x^2} + C$

$e^{-x^2}y(x) = -\frac{1}{2}e^{-x^2} + C$

$$\boxed{y(x) = -\frac{1}{2} + Ce^{+x^2}}$$

check: $y'(x) = 2xCe^{x^2}$

$y' - 2xy = 2xCe^{x^2} - 2x(-\frac{1}{2} + Ce^{x^2}) = x.$ ✓

Solutions for PS 2

pg 39 3. $y' + y = te^{-t} + 1$ Integrating factor

$p(t) = 1$ $\mu' = p\mu$

$g(t) = te^{-t} + 1$ $\mu' = \mu$ so $\mu = e^t$

$$\frac{d}{dt}(y\mu) = \frac{d}{dt}(e^t y) = e^t(te^{-t} + 1) = t + e^t$$

Integrate: $e^t y(t) = \int (t + e^t)\,dt = \frac{1}{2}t^2 + e^t + c$

$$\boxed{y(t) = \frac{1}{2}t^2e^{-t} + 1 + e^{-t}c}$$

Check: $y' = te^{-t} - \frac{1}{2}t^2e^{-t} - e^{-t}c$

$y' + y = te^{-t} + 1$ ✓

5. $y' - 2y = 3e^t$ $\mu' = -2\mu \Rightarrow \mu = e^{-2t}$

$p = -2$

$g = 3e^t$

$$\frac{d}{dt}(e^{-2t}y) = e^{-2t} \cdot 3e^t = 3e^{-t}$$

$\Rightarrow$ $e^{-2t}y = -3e^{-t} + c$

$$\boxed{y(t) = -3e^t + ce^{2t}}$$

Check: $y' = -3e^t + 2ce^{2t}$

$y' - 2y = 3e^t$ ✓

14. $y' + 2y = te^{-2t}$ $y(1) = 0$

$p = 2$ $\mu = e^{2t}$ (Check: $\mu' = 2\mu$)

$g = te^{-2t}$ $\frac{d}{dt}(e^{2t}y) = t \Rightarrow e^{2t}y = \frac{1}{2}t^2 + C$

PS2 (p2)

$$\boxed{y(t) = \tfrac{1}{2}t^2 e^{-2t} + Ce^{-2t}}$$

Check $y' = te^{-2t} - t^2e^{-2t} - 2Ce^{-2t}$

$y' + 2y = te^{-2t}$ ✓

Initial Cond.

$y(1) = 0 = \tfrac{1}{2}e^{-2} + Ce^{-2}$

$C\cancel{e^{-2}} = -\tfrac{1}{2}\cancel{e^{-2}}$ $C = -\tfrac{1}{2}$.

$$\boxed{y(t) = \tfrac{1}{2}t^2e^{-2t} - \tfrac{1}{2}e^{-2t}}$$

pg 47. 2. $y' = \dfrac{x^2}{y(1+x^3)}$ or $y\,dy = \left(\dfrac{x^2}{1+x^3}\right)dx$

let $u = 1+x^3$

$du = 3x^2dx$

$\tfrac{1}{2}y^2 + c = \tfrac{1}{3}\int \dfrac{du}{u} = \tfrac{1}{3}\ln|x^3+1|$

$$\boxed{\tfrac{1}{2}y^2 = \tfrac{1}{3}\ln|x^3+1| + c}$$

10. $y' = \dfrac{1-2x}{y}$ & $y(1) = -2$. $\int y\,dy = \int(1-2x)\,dx$

$\tfrac{1}{2}y^2 + C = x - x^2$

$y(1) = -2$ so $2 + C = 0$ $C = -2$

$$\boxed{\tfrac{1}{2}y^2 = 2 + x - x^2}$$

12. $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$ $r(1) = 2$: $\int \dfrac{dr}{r^2} = \int \dfrac{d\theta}{\theta}$ or $-r^{-1} + c = \ln|\theta|$

$-\dfrac{1}{r(1)} + C = \ln 1 = 0$ or $C = \dfrac{1}{2}$ $\boxed{\dfrac{1}{2} - \dfrac{1}{r} = \ln|\theta|}$

23. $y' = 2y^2 + \cancel{x}^2x = y^2(2+x)$ and $y(0) = 1$

$\dfrac{dy}{y^2} = (2+x)\,dx$ $-y^{-1} = 2x + \tfrac{1}{2}x^2 + C$

$-1 = C$ Critical pt: $y' = 0$

$\dfrac{-1}{2x + \frac{1}{2}x^2 - 1} = y(x)$ $\underline{x = -2}$ or $y = 0$