MA214–03 Spring 2009
Quiz #3 - 10 Points
6 February 2009

NAME: Solutions

1. (3 points). Find a solution of the initial value problem:

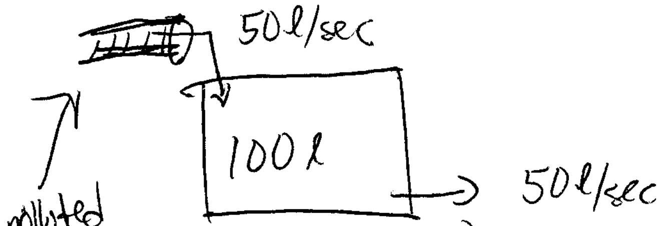
$$P'(t) = 2P(t), \quad P(t=0) = 4.$$

$P(t) = 4e^{2t}$ You should be able to write this!

$$\frac{dP}{P} = 2dt \qquad \ln|P| = 2t + C$$

$$\left.\begin{aligned} P(t) &= C_0 e^{2t} \\ P(t=0) &= C_0 = 4 \end{aligned}\right\} \quad P(t) = 4e^{2t}$$

2. (7 points). A full tank of 100 liters of polluted water initially contains 10 grams of pollutant. A solution pours into the tank at a rate of 50 *liters/sec* and has a concentration of pollutant given by 0.01*grams/liter*. The solution mixes uniformly in the tank and flows out at the same rate. Find the amount of pollutant in the tank as a function of t in seconds. How much pollutant is in the tank after 20 seconds?



$Q(t=0) = 10$ grams

$$\frac{dQ}{dt} = \begin{bmatrix} \text{flow of} \\ \text{pollutant} \\ \text{in} \end{bmatrix} - \begin{bmatrix} \text{flow of} \\ \text{pollutant out} \end{bmatrix}$$

$$= 50\frac{l}{sec} \cdot 10^{-2}\frac{g}{l} - \frac{Q(t)\,g}{100\,l} \cdot \frac{50\,l}{sec}$$

$$= \frac{1}{2}(1 - Q(t)) \ \ ^{l}/_{sec}$$

Solve: $\dfrac{dQ}{1-Q} = \dfrac{1}{2}dt$

$$\log|1 - Q| = -\frac{1}{2}t + C$$

$$Q(t) = C_0 e^{-t/2} + 1$$

Initial Condition:

$$Q(t=0) = C_0 + 1 = 10$$

$$C_0 = 9.$$

$$\boxed{Q(t) = 1 + 9e^{-t/2} \text{ grams}}$$

check $Q' = -\frac{9}{2}e^{-t/2} = \frac{1}{2}(-9e^{-t/2})$

$= \frac{1}{2}(-Q+1)$

$= \frac{1}{2}(1-Q)$ ✓

$Q(t=0) = 10$ grms

$$\boxed{Q(20) = 1 + 9e^{-10} \text{ gms}}$$