MA214–03 Fall 2008
Quiz #3 - 10 Points
22 September 2008

NAME: Solutions

1. (5 points). Find a solution of the initial value problem:

$$xy^2 + (4 + yx^2)y'(x) = 0, \quad y(0) = 1.$$

$M(x,y) = xy^2$
$N(x,y) = 4 + yx^2$

Exact? $\frac{\partial M}{\partial y} = 2xy = \frac{\partial N}{\partial x}$ Yes.

Solve for $\psi(x,y)$: $\frac{\partial \psi}{\partial x} = M$ & $\frac{\partial \psi}{\partial y} = N$

$\Rightarrow \psi = \frac{1}{2}x^2y^2 + h(y)$

$\psi = 4y + \frac{1}{2}y^2x^2 + g(x)$

We need $h(y) = 4y$ & $g(x) = 0$

Soln. $\psi = C = 4y + \frac{1}{2}y^2x^2$. Check: $0 = 4y' + yy'x^2 + xy^2$ ✓

$y(0) = 1 \Rightarrow C = 4$ so $\boxed{4 = 4y + \frac{1}{2}x^2y^2}$

2. (5 points). A tank holds 100 liters. A solution pours into the tank at a rate of 2 *liters/sec* and has a concentration of pollutant given by 0.01*grams/liter*. The solution mixes uniformly in the tank and flows out at the same rate. If the tank initially contains pure water, find the amount of pollutant in the tank as a function of t in seconds. How much pollutant is in the tank after 50*sec*?

$$\frac{dQ}{dt} = 2\frac{l}{s} \cdot 10^{-2}\frac{g}{l} - 2\frac{l}{s} \cdot \frac{Q(t)}{100}\frac{g}{l} = \frac{1}{50}\frac{g}{s} - \frac{Q(t)}{50}\frac{g}{s}$$

$Q(t) \equiv$ amount of pollutant in the tank at time t in grams.

$Q(t=0) = 0$.

Solve: $\frac{dQ}{dt} + \frac{1}{50}Q(t) = \frac{1}{50}$ $\quad p = \frac{1}{50}$ $\quad g = \frac{1}{50}$ (integrating factor)

$\mu(t) = e^{t/50}$

$Q(t)e^{t/50} = \frac{1}{50}\int e^{t/50}\,dt = e^{t/50} + C$

$Q(t) = 1 + Ce^{-t/50}$ $\quad Q(0) = 0 \Rightarrow C = -1$

$\boxed{Q(t) = 1 - e^{-t/50}}$

Check: $Q'(t) = \frac{1}{50}e^{-t/50}$

$Q' + \frac{1}{50}Q = \frac{1}{50}e^{-t/50} + \frac{1}{50} - \frac{1}{50}e^{-t/50} = \frac{1}{50}$ ✓

$\boxed{Q(50) = 1 - e^{-1}}$