MA214–03 Spring 2009
Quiz #4 - 10 Points
13 February 2009

NAME: Solutions

1. (4 points). Find the unique solution of the initial value problem for $x \geq 0$:

$$y' = \frac{x^4}{2y^3}, \quad y(1) = 1.$$

$2y^3\,dy = x^4\,dx$

$\frac{1}{2}y^4 = \frac{1}{5}x^5 + C$

$y^4 = \frac{2}{5}x^5 + C$

general solution

Initial Condition: $1 = \frac{2}{5} + C$

$C = \frac{3}{5}$

$x \geq 0$:

$y(x) = \sqrt[4]{\frac{2}{5}x^5 + \frac{3}{5}}$

2. (6 points). A person opens a bank account with an initial deposit of \$1,000. The account has a rate of return of 10%, compounded continuously, and the saver deposits \$1,000 continuously each year. How much is in the account after $(10 \ln 2)$ years? The ODE satisfied by the savings amount $S(t)$ at time $t \geq 0$ is

$$S'(t) = rS(t) + k.$$

$\int \frac{dS}{rS+k} = \int dt$

$u = rS + k$ substitute.

$du = r\,ds$

$\frac{1}{r}\log(rS+k) = t + C$

$rS + k = Ce^{rt}$

$\boxed{S(t) = Ce^{rt} - k/r}$

general solution

Intial Cond: $S(0) = 10^3$

$r = 0.1$

$k = 10^3$

$k/r = 10^4$

$10^3 = S(0) = C - 10^4$

$C = 1.1 \times 10^4 = 11{,}000$

$S(t) = 1.1 \times 10^4 e^{0.1t} - 10^4$

$S(10\ln 2) = 1.1 \times 10^4 e^{\ln 2} - 10^4$

$= 2.2 \times 10^4 - 10^4$

$= 1.2 \times 10^4$

$\boxed{S(10\ln 2) = 12{,}000 \text{ dollars}}$