MA214–03 Fall 2008
Quiz #4 - 10 Points
3 October 2008

NAME: SOLUTIONS

1. (5 points). Find the unique solution of the initial value problem:

$$y'' - y' - 2y = 0, \quad y(0) = 1, y'(0) = 0.$$

$r^2 - r - 2 = 0 = (r-2)(r+1)$ so $r_1 = 2$ $r_2 = -1$

General Soln: $y(t) = C_1 e^{2t} + C_2 e^{-t}$

$y'(t) = 2C_1 e^{2t} - C_2 e^{-t}$

$\{e^{2t}, e^{-t}\}$ are independent

Initial Conditions:

$y(0) = C_1 + C_2 = 1 \Rightarrow 3C_1 = 1$

$y'(0) = 2C_1 - C_2 = 0 \Rightarrow C_2 = 2C_1$

$C_1 = 1/3$ $C_2 = 2/3$

$\boxed{y(t) = \frac{1}{3}e^{2t} + \frac{2}{3}e^{-t}}$. Check: $y(0) = 1$ $y'(0) = 2/3 - 2/3 = 0$

2. (5 points). Find two solutions to the ODE

$$y'' - 2y' = 0,$$

and use the Wronskian to show that they are independent. What is the most general solution to this ODE?

$r^2 - 2r = r(r-2) = 0$ $r = 0$ $y_1(t) = 1$

$r = 2$ $y_2(t) = e^{2t}$

$W(y_1, y_2)(t) = \begin{vmatrix} 1 & e^{2t} \\ 0 & 2e^{2t} \end{vmatrix} = 2e^{2t} > 0$ never vanishes!

$\{y_1, y_2\}$ are independent.

General Soln $\boxed{y(t) = C_1 + C_2 e^{2t}}$.

Note: You can also guess $y(t) = 1$ is a soln.

If you let $u = y'$ the ODE $y'' - 2y' = u' - 2u = 0$

so $\frac{du}{u} = 2\,dt \Rightarrow \ln|u| = 2t$

$u(t) = y'(t) = Ce^{2t}$

so $y(t) = ce^{2t}$