NAME: SOLUTIONS

1. (5 points). Find the unique solution of the initial value problem:

\[ y'' - y' - 2y = 0, \quad y(0) = 1, y'(0) = 0. \]

\[ r^2 - r - 2 = 0 = (r - 2)(r + 1) \] so \( r_1 = 2 \) \( r_2 = -1 \)

General Soln: \( y(t) = C_1 e^{2t} + C_2 e^{-t} \)

\[ y'(t) = 2C_1 e^{2t} - C_2 e^{-t} \]

\( \{ e^{2t}, e^{-t} \} \) are independent

Initial Conditions:

\[ y(0) = C_1 + C_2 = 1 \rightarrow 3C_1 = 1 \rightarrow C_1 = \frac{1}{3} \quad C_2 = y_3 \]

\[ y'(0) = 2C_1 - C_2 = 0 \rightarrow C_2 = 2C_1 \rightarrow y_3 = \frac{2}{3} \]

\[ y(t) = \frac{1}{3} e^{2t} + \frac{2}{3} e^{-t} \]

Check: \( y(0) = 1 \quad y'(0) = \frac{1}{3} - \frac{2}{3} = 0 \)

2. (5 points). Find two solutions to the ODE

\[ y'' - 2y' = 0, \]

and use the Wronskian to show that they are independent. What is the most general solution to this ODE?

\[ r^2 - 2r = r(r - 2) = 0 \quad r = 0 \quad y_1(t) = 1 \]

\[ r = 2 \quad y_2(t) = e^{2t} \]

\[ W(y_1, y_2)(t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = e^{2t} > 0 \text{ never vanishes}. \]

\( \{ y_1, y_2 \} \) are independent.

General Soln \( y(t) = C_1 + C_2 e^{2t} \).

Note: You can also guess \( y(t) = 1 \) is a soln.
If you let \( u = y \), the ODE \( y'' - 2y' = y'' - 2u = 0 \)

so \( \frac{du}{dt} = 2dt \rightarrow \ln |u| = 2t \)

\( u(t) = y'(t) = e^{2t} \)

so \( y(t) = e^{2t} \)