MA214–03 Spring 2009
Quiz #5 - 10 Points
20 February 2009

NAME: Solutions

1. (6 points). Find the unique solution of the initial value problem:

$$x \sin y + \left(\frac{1}{2}x^2 \cos y\right) y' = 0, \quad y(1) = \frac{\pi}{2}.$$

$M(x,y) = x\sin y \quad \frac{\partial M}{\partial y} = x\cos y \quad N(x,y) = \frac{1}{2}x^2\cos y \quad \frac{\partial N}{\partial x} = x\cos y$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ the ODE is exact.

Solve $\frac{\partial \psi}{\partial x} = M$ so $\psi(x,y) = \int M(x,y)\,dx = \frac{1}{2}x^2\sin y + g(y)$

$\frac{\partial \psi}{\partial y} = N$ so $\psi(x,y) = \int N(x,y)\,dy = \frac{1}{2}x^2\sin y + h(x)$ $\Big\}$ $g = h = 0$

$\psi(x,y) = \boxed{\frac{1}{2}x^2 \sin y = C}$ Set $x=1$: $\frac{1}{2}(1)^2 \sin\frac{\pi}{2} = \frac{1}{2} = C$

$\boxed{\text{Soln: } x^2 \sin y = 1}$

2. (4 points). Find the most general solution of the second-order ODE:

$$y'' - y' - 6y = 0.$$

Characteristic quadratic eqn: $r^2 - r - 6 = 0$
Obtain by setting $y(t) = e^{rt}$ & substituting into the ODE.

$r^2 - r - 6 = (r-3)(r+2) = 0 \quad y_1(t) = e^{3t} \quad y_2(t) = e^{-2t}$

General Solution:

$$\boxed{y(t) = C_1 e^{3t} + C_2 e^{-2t}}$$