

NAME: Solutions

1. (6 points). Find the most general solution to the ODE:

$$y'' + 2y' + y = 0.$$

Find the unique solution to this ODE with the initial conditions: $y(0) = 1$ and $y'(0) = 1$.

$$r^2 + 2r + 1 = (r+1)^2 = 0 \quad r = -1 \text{ repeated root}$$

General Soln.: $y(t) = C_1 e^{-t} + C_2 t e^{-t}$

Initial Cond.: $y'(t) = -C_1 e^{-t} + C_2 e^{-t} - C_2 t e^{-t}$

$$y(0) = C_1 = 1$$

$$y'(0) = -1 + C_2 = 1 \text{ so } C_2 = 2$$

$$\left\{ \begin{array}{l} y(t) = e^{-t} (1 + 2t) \end{array} \right.$$

2. (4 points). Write each of the following complex numbers in the form $x + iy$, with x and y real.

a) 3^{2+i} ,

b) $e^{(\pi/2+i)i}$.

$$a) 3^{2+i} = 3^2 e^{i \ln 3} = (9 \cos(\ln 3)) + i(9 \sin(\ln 3))$$

$$b) e^{(\pi/2+i)i} = e^{\pi/2 i} e^{-1} = e^{-1} (\cos \pi/2 + i \sin \pi/2) = i e^{-1}$$

$$a) 9 \cos(\ln 3) + i(9 \sin(\ln 3))$$

$$b) i e^{-1}$$