MA214–03 Spring 2009
Quiz #6 - 10 Points
13 March 2009

NAME: Solutions

1. (5 points). Find the unique solution to the initial value problem for the ODE:

$$y'' + 4y' + 4y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

$r^2 + 4r + 4 = 0 = (r+2)^2 = 0 \quad r = -2$ repeated root

2 independent solns: $y_1(t) = e^{-2t} \quad y_2(t) = te^{-2t}$

General soln: $y(t) = C_1 e^{-2t} + C_2 te^{-2t}$

$y'(t) = -2C_1 e^{-2t} + C_2 e^{-2t} - 2C_2 te^{-2t}$

Initial Conditions: $y(0) = C_1 = 0$

$y'(0) = C_2 = 1$

Soln: $\boxed{y(t) = te^{-2t}}$

2. (5 points). Find two real independent solutions of the ODE:

$$y'' + 2y' + 2y = 0.$$

$r^2 + 2r + 2 = 0$, roots: $\dfrac{-2 \pm [4-8]^{\frac{1}{2}}}{2} = -1 \pm i$

2 indep. real solns:

$$\boxed{\begin{aligned} y_1(t) &= e^{-t}\cos t \\ y_2(t) &= e^{-t}\sin t \end{aligned}}$$

Note that $w_1(t) = e^{(-1+i)t} = e^{-t}(\cos t + i\sin t)$

$w_2(t) = e^{(-1-i)t} = e^{-t}(\cos t - i\sin t) = \overline{w_1}(t)$

so $y_1(t) = \frac{1}{2}(w_1(t) + \overline{w_1}(t)) = \mathrm{Re}\, w_1(t)$

$y_2(t) = \frac{1}{2i}(w_1(t) - \overline{w_1}(t)) = \mathrm{Im}\, w_1(t)$.