**MA214–03 Fall 2008**
**Quiz #6 - 10 Points**
**31 October 2008**

NAME: Solutions

1. (5 points). Use the method of undetermined coefficients to find the general solution of the ODE:

$$y'' + 4y = 2.$$

homog. ODE : $y'' + 4y = 0$ $\quad y_{homog}(t) = C_1 \cos 2t + C_2 \sin 2t$

particular soln: Guess $y_p = A + Bt + Ct^2$

$y_p' = B + 2Ct$

$y_p'' + 2C$

Substitute: $2C + 4A + 4Bt + 4Ct^2 = 2$ $\quad B = 0$, $C = 0$, $A = \frac{1}{2}$

$y_p(t) = \frac{1}{2}$ $\quad$ check: $y_p'' + 4y_p = 0 + 4 \cdot \frac{1}{2} = 2.$

General Soln: $y(t) = \frac{1}{2} + C_1 \cos 2t + C_2 \sin 2t$

2. (5 points). Use the variation of parameters method and find the most general solution of the ODE:

$$y'' - 3y' + 2y = e^t.$$

homog. ODE: $\left.\begin{array}{l} r^2 - 3r + 2 = 0 \\ (r-1)(r-2) = 0 \end{array}\right\}$ $\quad y_{homog}(t) = C_1 e^t + C_2 e^{2t}$

$y_1(t) = e^t \quad y_2(t) = e^{2t}$

$$W(y_1, y_2)(t) = \begin{vmatrix} e^t & e^{2t} \\ e^t & 2e^{2t} \end{vmatrix} = e^{3t}.$$

particular soln:

$u_1' = \frac{-g y_2}{W} = -\frac{e^t \cdot e^{2t}}{e^{3t}} = -1 \qquad u_1(t) = -t$

$u_2'(t) = \frac{g y_1}{W} = \frac{e^t \cdot e^t}{e^{3t}} = e^{-t} \qquad u_2(t) = -e^{-t}$

$y_p(t) = (u_1 y_1)(t) + (u_2 y_2)(t) = -te^t - e^t$

note that $-e^t$ is a soln to the homog. problem so we take

$y(t) = C_1 e^t + C_2 e^{2t} - te^t. \quad y_p(t) = -te^t$

Check: $\left.\begin{array}{l} y_p'(t) = -e^t(1+t) \\ y_p''(t) = -e^t(2+t) \end{array}\right\}$ $(-2 - t + 3(1+t) - 2t)e^t \overset{?}{=} e^t$ yes!