MA214-03 Spring 2009
Quiz #7 - 10 Points
27 March 2009

NAME: Solutions

1. (7 points) Find the most general solution to the ODE:

$$y'' - y = 2e^{2t}.$$

2. (3 points) Find the unique solution to this ODE satisfying the initial conditions:

$$y(0) = 1, \quad y'(0) = 1.$$

homog ODE $\quad y'' - y = 0 \qquad r^2 - 1 = 0 \quad r_1 = +1, \ r_2 = -1$

$y_1(t) = e^t \qquad y_2(t) = e^{-t}$

$W(y_1, y_2)(t) = \begin{vmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{vmatrix} = -2 \neq 0.$

$\boxed{y_{homog}(t) = C_1 e^t + C_2 e^{-t}}$

nonhomog. ODE $\quad$ particular soln.

Method 1 $\quad$ undeter. coef. $\quad$ guess $\quad Ae^{2t} = y_p(t)$

$y_p'(t) = 2Ae^{2t}$

$y_p''(t) = 4Ae^{2t}$

subst. $\quad (4A - A)e^{2t} = 2e^{2t} \qquad A = 2/3 \qquad \boxed{y_p(t) = \tfrac{2}{3} e^{2t}}$

Method 2 $\quad$ Variation of param.

$c_1' = -\dfrac{y_2 g}{W} = -\dfrac{e^{-t} \cdot 2e^{2t}}{-2} = e^t \Rightarrow c_1(t) = e^t$

$c_2' = \dfrac{y_1 g}{W} = \dfrac{e^t \cdot 2e^{2t}}{-2} = -e^{3t} \Rightarrow c_2 = -\frac{1}{3} e^{3t}$

$y_p(t) = e^t \cdot e^t + \left(-\frac{1}{3} e^{3t}\right) e^{-t} = \frac{2}{3} e^{2t}.$

General Soln: $\boxed{y(t) = C_1 e^t + C_2 e^{-t} + \tfrac{2}{3} e^{2t}}$

Initial Cond: $\quad y'(t) = C_1 e^t - C_2 e^{-t} + \frac{4}{3} e^{2t} \qquad y'(0) = C_1 - C_2 + \frac{4}{3} = 1$

$y(0) = C_1 + C_2 + \frac{2}{3} = 1$

$2C_1 + 2 = 2 \qquad C_1 = 0$

$2C_2 - \frac{2}{3} = 0 \qquad C_2 = \frac{1}{3} \qquad \boxed{y(t) = \tfrac{1}{3} e^{-t} + \tfrac{2}{3} e^{2t}}$