MA214–03 Fall 2008
Quiz #7 - 10 Points
7 November 2008

NAME: Solutions

1. (5 points). Consider the ODE for an oscillator at resonance:

$$y'' + y = 2\cos t.$$

Show that $y_p(t) = t\sin t$ is a particular solution. Write the most general solution of this ODE.

Check $y_p'(t) = \sin t + t\cos t$ } $y_p'' + y_p = 2\cos t$ ✓
$y_p''(t) = 2\cos t - t\sin t$

Homog. ODE: $y'' + y = 0$ $\quad C_1\cos t + C_2\sin t$ general soln.

$$\boxed{y(t) = C_1\cos t + C_2\sin t + t\sin t}$$

2. (5 points). Solve the initial value problem:

$$y'' + 2y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

$r^2 + 2r + 5 = 0$ $\quad$ 2 roots: $\dfrac{-2 \pm [4-20]^{\frac{1}{2}}}{2} = -1 \pm 2i$

Real Solns: $e^{-t}\cos 2t$, $e^{-t}\sin 2t$ $\quad$ (use Euler: $e^{2it} = \cos 2t + i\sin 2t$)

General Soln:

$y(t) = C_1 e^{-t}\cos 2t + C_2 e^{-t}\sin 2t$ $\qquad y(0) = C_1 = 0$

$y'(t) = -C_2 e^{-t}\sin 2t + 2C_2 e^{t}\cos 2t$

$y'(0) = 2C_2 = 1 \quad C_2 = \frac{1}{2}$

so $\boxed{y(t) = \frac{1}{2}e^{-t}\sin 2t}$