MA214–03 Spring 2009
Quiz #8 - 10 Points
3 April 2009

NAME: Solutions

1. Find the unique solution to the driven oscillator ODE:

$$u''(t) + 4u(t) = \cos(2t),$$

with initial conditions $u(0) = 0 = u'(0)$.

Trig identities that you might need:

$$cos^2 t = (1/2)(1 + \cos(2t)),$$

$$sin^2 t = (1/2)(1 - \cos(2t)).$$

Reduce $\sin(4t)$ using:

$$\sin 2t = 2 \sin t \cos t.$$

homog. ODE: $u_1(t) = \cos 2t \qquad u_2(t) = \sin 2t$

$$W = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix} = 2$$

Variation of Param: $c_1' = -\frac{g u_2}{W} = -\frac{1}{2} \cos 2t \sin 2t \Rightarrow c_1(t) = -\frac{1}{8} \sin^2 2t$

$u = \sin 2t$
$du = 2\cos 2t$

$$c_2' = \frac{g u_1}{W} = \frac{1}{2} \cos^2 2t \Rightarrow c_2 = \frac{1}{4} \int (1 + \cos 4t) = \frac{1}{4} t + \frac{1}{16} \sin 4t$$

$$u_p(t) = -\frac{1}{8} \sin^2 2t \cos 2t + \frac{1}{4} t \sin 2t + \frac{1}{16} \sin 4t \sin 2t$$

$$= \frac{1}{4} t \sin 2t + \frac{1}{8} \sin^2 2t \cos 2t - \frac{1}{8} \sin^2 2t \cos 2t = \frac{1}{4} t \sin 2t$$

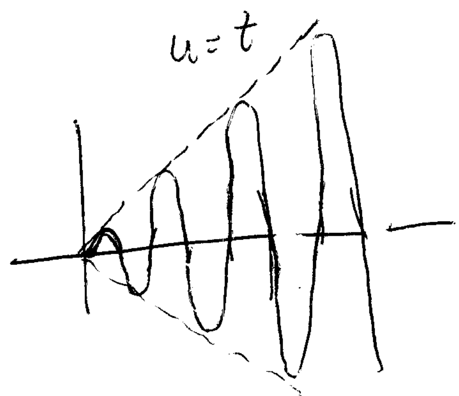
General soln: $u(t) = c_1 \cos 2t + c_2 \sin 2t + \frac{1}{4} t \sin 2t$

Initial Conditions: $u'(t) = -2c_1 \sin 2t + 2c_2 \cos 2t + \frac{1}{4} \sin 2t + \frac{1}{2} t \cos 2t$

$u(0) = 0 = c_1$

$u'(0) = 2c_2 = 0.$

$$\boxed{u(t) = \frac{1}{4} t \sin 2t}$$



Resonant growth