

NAME: _____

Solutions

1. Use the Laplace transform method to find the unique solution ODE:

$$u''(t) + 4u(t) = 2,$$

with initial conditions $u(0) = 1$ and $u'(0) = 0$. Formulas are on the board.Step 1

$$(s^2 + 4)(\mathcal{L}u)(s) - s = \frac{2}{s}$$

$$(\mathcal{L}u)(s) = \frac{2}{s(s^2+4)} + \frac{s}{s^2+4}$$

Step 2

Partial Fraction

$$\frac{2}{s(s^2+4)} = \frac{As+B}{s^2+4} + \frac{C}{s}$$

irreducible quadratic

$$2 = As^2 + Bs + Cs^2 + 4C$$

$$B = 0$$

$$C = \frac{1}{2} \quad A = -\frac{1}{2}$$

$$A + C = 0$$

$$2 = 4C$$

$$(\mathcal{L}u)(s) = -\frac{1}{2} \frac{s}{s^2+4} + \frac{1}{2s} + \frac{s}{s^2+4} = \frac{1}{2} \frac{s}{s^2+4} + \frac{1}{2s}$$

$$u(t) = \frac{1}{2} \cos 2t + \frac{1}{2}$$

1

$$u(0) = \frac{1}{2} + \frac{1}{2} = 1$$

$$u'(0) = 0$$