MA214–03 Fall 2008
Quiz #9 - 10 Points
21 November 2008

NAME: Solutions

FORMULAS ON THE BOARD

1. (4 points). Find the function $f(t)$ that has the Laplace transform:

$$(\mathcal{L}f)(s) = \frac{e^{-3s}}{s^2+2s+5}. = e^{-3s} H(s)$$

$$H(s) = \frac{1}{s^2+2s+5} = \frac{1}{(s+1)^2+4} = \frac{1}{2}\frac{2}{(s+1)^2+2^2}$$

$$(\mathcal{L}^{-1}H)(t) = \frac{1}{2}e^{-t}\cos 2t$$

$$f(t) = u_3(t)(\mathcal{L}^{-1}H)(t-3) = \frac{1}{2}u_3(t)e^{-(t-3)}\cos 2(t-3)$$

$$\boxed{f(t) = \frac{1}{2}u_3(t)e^{-(t-3)}\cos 2(t-3)}$$

2. (6 points). Find the unique solution $y(t)$ of the initial value problem:

$$y'' + y = u_2(t), \quad y(0) = y'(0) = 0.$$

$$(s^2+1)(\mathcal{L}y)(s) = \frac{e^{-2s}}{s} \quad \text{so} \quad (\mathcal{L}y)(s) = e^{-2s}H(s)$$

$$H(s) = \frac{1}{s(s^2+1)} = \frac{As+B}{s^2+1} + \frac{C}{s} \quad \text{so} \quad 1 = As^2 + Bs + Cs^2 + C$$

$$C = 1 \quad B = 0 \quad A = -C = -1$$

$$H(s) = -\frac{s}{s^2+1} + \frac{1}{s}$$

$$(\mathcal{L}^{-1}H)(t) = -\sin t + 1$$

$$y(t) = u_2(t)(\mathcal{L}^{-1}H)(t-2) = u_2(t)(1-\sin(t-2))$$

$$\boxed{y(t) = u_2(t)(1-\sin(t-2))}$$