MA/PHY506 Fall 2012
Problem Set 6
DUE: 12 November 2012

1. Arfken, Chapter 8, page 388, problem 8.2.10.

2. Let ϕ_j be an orthonormal basis of the Hilbert space $L^2([a,b])$. For any $f \in L^2([a,b])$, the mean square (MS) error between f and the finite series approximation $S_N(x) = \sum_{j=1}^{N} c_j \phi_j(x)$ is defined by

$$MS_N(f;\{c_j\}) \equiv \int_a^b |f(x) - S_N(x)|^2 \, dx.$$

   Assume that f, the coefficients c_j, and the basis functions ϕ_j are all real (for simplicity). Show that $MS(f;\{c_j\})$ is minimized with the choice $c_j = \int_a^b \phi_j(x) f(x)\, dx$, the expansion coefficients of f relative to the orthonormal basis ϕ_j.

3. Consider the nonhomogeneous BVP: $y'' = x(x-2\pi)$ on $[0,\pi]$. Expand y in the eigenfunctions of the related Sturm-Liouville problem $Ly = -y'' = \lambda y$ with DBC at 0 and π. Expand $h(x) = x(x-2\pi)$ in the eigenfunctions of this Sturm-Liouville problem. Find a formal series solution for y.

4. Arfken, Chapter 5: problems 5.1.1, 5.1.5 part a.