1. Arfken, Chapter 8, page 388, problem 8.2.10.

2. Let $\phi_j$ be an orthonormal basis of the Hilbert space $L^2([a, b])$. For any $f \in L^2([a, b])$, the mean square (MS) error between $f$ and the finite series approximation $S_N(x) = \sum_{j=1}^{N} c_j \phi_j(x)$ is defined by

$$MS_N(f; \{c_j\}) \equiv \int_a^b |f(x) - S_N(x)|^2 \, dx.$$  

Assume that $f$, the coefficients $c_j$, and the basis functions $\phi_j$ are all real (for simplicity). Show that $MS(f; \{c_j\})$ is minimized with the choice $c_j = \int_a^b \phi_j(x) f(x) \, dx$, the expansion coefficients of $f$ relative to the orthonormal basis $\phi_j$.

3. Consider the nonhomogeneous BVP: $y'' = x(x-2\pi)$ on $[0, \pi]$. Expand $y$ in the eigenfunctions of the related Sturm-Liouville problem $Ly = -y'' = \lambda y$ with DBC at $0$ and $\pi$. Expand $h(x) = x(x-2\pi)$ in the eigenfunctions of this Sturm-Liouville problem. Find a formal series solution for $y$.

4. Arfken, Chapter 5: problems 5.1.1, 5.1.5 part a.