

MA/PHY506 Fall 2015

Problem Set 6

DUE: 16 October 2015

1. Apply the Gram-Schmidt method to the functions $f_0(t) = 1, f_1(t) = t, f_2(t) = t^2$ on the interval $[-1, 1]$ in order to obtain an orthonormal set of functions. The inner product is

$$\langle f, g \rangle = \int_{-1}^1 f(t)g(t) dt.$$

Compare these with the first three Legendre polynomials in the text.

2. What is the dimension of the subspace of R^3 spanned by the vectors: $(2, 1, -1), (3, 2, 1), (1, 0, -3)$? What is a general condition so that n -vectors in R^n are linearly independent?
3. Let $\{v_j \mid j = 1, \dots, K\}$ be a finite orthonormal set in an inner product space V . For any vector $v \in V$, show that

$$\sum_{j=1}^K |(v, v_j)|^2 \leq \|v\|^2.$$

This is called Bessel's inequality.

4. Arfken, page 393, problems 8.3.1 and 8.3.4.